Firm-level productivity and demand shocks in imperfectly competitive labor markets: implications for wage dynamics*

Micole De Vera†
CEMFI

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Abstract

I build a novel framework to empirically disentangle productivity from demand shocks at the firm level and measure their pass-through to worker wages. Measuring the pass-through of firm-level shocks contributes to understanding how firms affect wage inequality and wage dynamics. My analysis leverages a unique Portuguese data set that combines matched employer-employee data, financial statements data, and firm-product information on quantities and prices. The productivity, demand, and labor market advantages processes are inferred from observed data. I find substantial cross-sectional heterogeneity across firms along these three dimensions. Moreover, these features of the firm evolve in rich, nonlinear ways. Most firms have highly persistent states for most shocks, but poor-performing firms with large positive shocks have much less persistent states. In an environment with wage adjustment costs, I find that wages are not adjusted in response to productivity shocks, whereas I estimate positive pass-through elasticities to demand shocks. There is suggestive evidence that pass-throughs of adverse demand shocks are larger than those of good ones. Moreover, the pass-through of good demand shocks is larger for firms with better positions in their respective labor markets.

JEL codes: C33, D24, J31, J42, L11

Keywords: wage pass-through, nonlinear firm dynamics, imperfect competition, nonlinear panel data models, Portugal

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†E-mail address: micole.devera@cemfi.edu.es; Website: micoledevera.github.io

1 Introduction

There has been a recent renewed and growing interest in measuring labor market power owing to the rise of access to large administrative data that match employee data with employer information. A large body of empirical work using matched employer-employee data that has stemmed from the seminal work by Abowd et al. (1999) has shown that firms play a role in explaining wage heterogeneity. Measuring the pass-through of idiosyncratic firm shocks is a way to better understand the ways in which firms affect not only wage inequality, but also wage dynamics. Due to limitations in most available data sets, papers that measure wage pass-through have focused on revenue or value-added shocks (e.g., Guiso et al., 2005) or relied on parametric structural models to disentangle productivity and demand shocks (e.g., Lamadon, 2016).[1] Disentangling the pass-through of productivity from that of demand shocks not only gives us a better picture of how firms use their market power, but also a better understanding of the interaction of technology, the product market, and the labor market.

In this paper, I measure whether persistent idiosyncratic shocks to firm productivity or demand are translated to adjustment in wages, and see whether it is mediated by labor market power. The main challenge in estimating the pass-through of productivity and demand shocks, and understanding how they differ based on labor market power of the firm, is that these objects are unobserved to us but are crucial state variables when the firm makes decisions, including wage-setting. Therefore, I develop and estimate a flexible, semi-structural framework of firms and workers to disentangle different shocks that firms face and understand how these affect the decisions of the firm, particularly in wage-setting. To be internally consistent, firms in my model operate in an environment with imperfect competition in both the output and input markets.

This framework leverages a unique Portuguese data set that combines a matched employer-employee data set, financial statements data, and a firm-product level manufacturing survey that includes quantity and price information at the product level. The availability of price and quantity data at the firm-product level allows me to flexibly disentangle productivity from demand advantages. My empirical fo-

[1] Separate strands of the literature used reduced-form and quasi-experimental methods to study the labor market effects of productivity shocks (e.g., Kátay, 2016; Chan et al., 2019) and demand shocks (e.g., Garin and Silverio, 2018). I subsequently provide a more extensive review of the related literature.

cus is on large firms in three particular manufacturing sectors in Portugal between 2012 and 2018, which covers about 39% of total manufacturing employment and 24% of total manufacturing sales.[2]

Main results. The main contributions of this paper are twofold: methodological and empirical. *First*, I propose a novel empirical framework to estimate productivity and demand advantages in the presence of imperfect competition in both the output and input markets, as well as wage and capital adjustment costs. I propose a simulation-based algorithm to estimate the model. As mentioned, this approach is only possible because of a unique data set that combines matched employer-employee data with financial statement data and firm-product specific data on quantities and prices. On the one hand, the availability of matched employer-employee data provides additional information to control for worker heterogeneity affecting wages. On the other hand, availability of quantity and price data allows us to disentangle flexibly variations in productivity and demand.

A key feature of this model is its flexibility which makes it appropriate as a tool to measure important features of the environment and decisions of the firm. There are two dimensions in which the flexibility of the model is crucial for the question of interest. First, I allow for flexible dynamics in firm heterogeneity. Productivity, consumer preferences, and labor market power likely evolve in rich, dynamic ways. For instance, an initially poor-performing firm may luckily hire an enthusiastic manager who overhauls the production process and serves as a good mentor to new employees. This has an effect of boosting the productivity of the firm, and increasing the attractiveness of the firm as an employer as workers would want to benefit from good mentorship and a good working climate. Formally, the arrival of the positive shock wipes out the persistence of the existing poor state of the firm. Alternatively, the products of a well-respected firm might fall in favor because of new research of its adverse health effects—which not only shifts consumer preferences away from the firm's product, but also workers may actively avoid the firm with concern of the health effects of producing the products. In this case, a negative shock can erase the good history of a well-positioned firm. The model I build is able to capture such kinds of dynamics by (i) specifying flexible dynamics of firm heterogeneity where the persistence of a state may depend on the current

[2] Data on all other manufacturing sectors are available. The sectors I consider are the three largest in terms of number of firms. I focus on these sectors only for computational tractability.

shock and the state itself (Arellano et al., 2017), and (ii) allowing the shocks to productivity, demand, and labor market position to be arbitrarily correlated.

The second dimension of flexibility comes from estimating empirical policy functions of the firms which are agnostic to the specification of some structural objects. To be concrete, I estimate a wage-setting equation that allows for wage adjustment costs without having to specify the specific form of the wage adjustment cost. We do not want our measurement to depend on the parametrization of structural objects and so allowing for flexibility in that regard allows to be more confident in the relationships that we measure.[3]

Second is an empirical contribution: using estimates from the estimated model, I present findings on three dimensions of firm heterogeneity which I call productivity, demand advantages, and labor market advantages. Productivity measures the efficiency by which firms convert inputs into outputs and is modeled as a conventional factor-neutral term in the production function. Demand advantages, on the other hand, capture horizontal product differences across firms—these are captured by differences in output demand holding prices fixed. Lastly, differences in labor market advantages capture differences in the labor supply elasticities faced by firms which may arise, for instance, from differences in amenities, which are non-pecuniary attributes of the firm that changes the attractiveness of firms to workers (e.g., Card et al., 2018).

The quantitative results of the model on these three dimensions of firm heterogeneity relate to their (i) cross-sectional distribution; (ii) dynamics; and (iii) pass-through to worker wages.

I find that there is substantial cross-sectional heterogeneity across firms. Fixing inputs, the firm a the 90th percentile of the productivity distribution produces 8 times more than the firm at the 10th percentile of the distribution. This is comparable to previous estimates of the productivity dispersion using alternative data sets and estimation methods. Moreover, I find that holding prices constant, there are large differences in output demand where the firm at the 90th percentile of the demand advantage distribution has a demand that is about 72 times greater than the firm at the 10th percentile.

[3]The model has a modular structure and can be extended to study other aspects of the firms such as decisions on worker composition or on investment in research and development. A cost of being flexible in this manner is that we are constrained by the kinds of counterfactuals which we can measures, specifically those that do not change the policy functions of the firm.

With respect to the labor market, I find that firms differ in the labor supply elasticity they face. At the median, the labor supply elasticity with respect to wages is about 0.6 but this ranges from 0.3 at the 10th percentile to 1.2 at the 90th percentile. Under static wage-setting, i.e., without wage adjustment costs, this translates into substantial markdowns where wages are 0.25–0.55 of marginal revenue product of labor. In the presence of adjustment costs, we would expect that wages are a larger share of marginal product to compensate for the static adjustment costs and the dynamic value of wages as they affect future wage setting.

I find that there is a strong positive correlation between productivity and demand advantages, as well as, demand and labor market advantages. This is partially accounted for by dependence in the shocks—productivity and demand shocks as well as demand and labor market advantage shocks are positively correlated—and partly by dependence in initial states. I also find that the distribution of state changes depend on the position of the firm in the overall distribution. In particular, firms at the bottom of the productivity distribution face more uncertainty that is skewed towards good shocks. On the other hand, firms at the top of the productivity distribution face less uncertainty but this uncertainty is skewed towards bad shocks. This would suggest that there is more room for improvement at the bottom of the distribution and more room to fall from the top of the distribution. Similar patterns are seen in demand and labor market advantages.

This has important implications for the modeling of the dynamics of firm heterogeneity. Often, empirical models of productivity or demand parametrize the dynamics of firm heterogeneity as linear autoregressive processes with Gaussian errors. Such a model would suggest that the level of uncertainty faced by firms is constant and that there is no asymmetry in the distribution of shocks. My results suggest that these models are counterfactual and miss substantial asymmetries in the dynamics of firm heterogeneity that will affect firm decisions.

Overall, I find that the wage effects of productivity and demand shocks are (i) asymmetric and (ii) depend on the level of labor market advantages. I find the wage pass-through elasticities of productivity shocks to be close to zero. On the other hand, I estimate wage pass-through elasticities of demand shocks of around 0.02 for bad demand shocks and 0.01 for good demand shocks. These estimates are smaller compared to estimates of the pass-through of revenue and value-added shocks which range between 0.06–0.09 (Guiso et al., 2005; Cardoso and Portela, 2009). The pass-through estimates I estimate differ from those in the literature using revenue or value-added along multiple dimensions. Two substantive differences are (i) I estimate distinct pass-through parameters for productivity and demand shocks separately, and (ii) the framework I build accommodates for the presence of wage adjustment costs.

The pass-through of bad demand shocks does not seem to be mediated by the position of the firm in the labor market advantage distribution. On the other hand, the pass-through of bad demand shocks are larger in firms which are at the top of the labor market distribution. In fact, wages at the median of the labor market advantage distribution face a pass-through elasticity of about 0.02 with respect to a good demand shock. On the other hand, the firm at the 10th percentile of the labor market advantage distribution does not adjust wages faced with the same good demand shock.

Related literature and additional contributions. I highlight important contributions this paper makes to three particular strands of the literature. *First*, this paper adds to the literature studying the wage pass-through of firm-level shocks using microdata. A number of papers have studied the pass-through of sales or value-added to worker wages using microdata starting with the seminal paper of Guiso et al. (2005) using Italian matched employer-employee data. They specify a stochastic permanent-transitory structure to value-added and specify a wage determination equation, then estimate the parameters by matching the covariance structures. They find that firms provide full insurance against temporary shocks and partial insurance against permanent shocks to firm performance. The same has been found in Portugal using a similar methodology (Cardoso and Portela, 2009). On the other hand, Guertzgen (2014) finds that worker wages are also fully insured against permanent shocks in Germany. Minimal wage transmission of shocks to firm revenue was also found in the United States (Juhn et al., 2018).

Shocks to sales or value-added are a composition of different shocks that the firm faces (e.g., productivity shocks and demand shocks, to name two specific ones). Another strand of this literature has focused on isolating the pass-through of productivity shocks on wages. Primarily, two methods have been used to isolate productivity shocks and study its pass-through to wages. On the one hand, there is the more "reduced-form" strategy which combines production function estimation to recover productivity shocks (such as proxy methods as in Olley and Pakes, 1996, and its extensions) with covariance structures to estimate its pass-through to wages (e.g., as in Guiso et al., 2005, or VARs). Some papers using this strategy include work by Kátay (2016) for Hungary and Chan et al. (2019) for Denmark. On the other hand, there are the papers which build and estimate structural models to disentangle productivity and measure its pass-through (Lamadon, 2016; Friedrich et al., 2019; Lamadon et al., 2019).

A complementary literature studies the pass-through of product demand shocks or shocks to product market competition to workers. This strand of the literature has previously focused on shocks to entire industries or sectors (Abowd and Lemieux, 1993; Guadalupe, 2007; Verhoogen, 2008). With the availability of microdata, a few papers have focused on idiosyncratic shocks. Garin and Silverio (2018) argue the quasi-randomness of import demand shifts to isolate the effect of idiosyncratic product demand shocks on wages in Portuguese firms. Cho (2018) compares workers in firms affected by the American Recovery and Reinvestment Act, implemented during the Great Recession, to workers in firms not connected to the act. These two papers find that firms that experience positive demand shocks increase both employment and wages.

In contrast to the above mentioned papers, I measure the wage pass-through of both productivity and demand shocks to the firm in a unified framework. Moreover, I explore nonlinearities in the wage pass-through of these shocks by allowing the wage pass-through elasticity to flexibly depend on the level of productivity, demand advantages, and labor market advantages.

Second, this paper relates to identification and estimation of production functions and productivity, particularly in the presence of imperfect competition in output or input markets.[4] In my proposed framework, we allow for dynamic, latent demand advantage heterogeneity reflecting horizontal differentiation in the economy. Currently popular methods to estimate the production function (and, consequently, productivity) using firm microdata have been developed under a framework of competitive input and output markets. Competitive output markets is an underlying identifying assumption in the so-called proxy method or control function approaches to production function estimation (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg et al., 2015). The bias that arises as a result of unobserved prices or demand advantages has been studied by Klette and Griliches (1996). This output price bias spills over to the measurement of other

[4] De Loecker and Syverson (2021) provides a current review of the issues on productivity — both its measurement and analysis.

quantities relying on good estimates of the production function; for instance, in productivity estimation or in price markup estimation using the methodology proposed by De Loecker and Warzynski (2012) (Bond et al., 2021). Even if we are willing to estimate a "composite" object that combines both productivity and demand advantages using classical methods and revenue data, Guillard et al. (2018) argues that the estimated composite may not behave as economically meaningful especially if productivity and demand heterogeneity are correlated.

From the point of view of production function estimation, demand advantages and output price variation are often considered nuisances. With the increasing availability of detailed firm-level microdata, a popular way to deal with this is the use of firm-specific price indices to deflate revenues into real terms (Eslava et al., 2004; Smeets and Warzynski, 2013; Carlsson et al., 2016). For these papers, these methods may also double to address the multi-product nature of firms. In this paper, I use a similar method to separate real output and prices. However, as both productivity and demand advantages are both objects of interest, I model both real output and prices jointly.

This is not the first paper that has interest in identifying both productivity and demand shocks separately. In their influential paper, Foster et al. (2008) argue that productivity investments and shifts in the demand curve are two relevant dimensions which helps us better understand firm growth, entry and exit.[5] They are able to disentangle productivity and demand shifts by focusing on quasi-homogeneous goods where quality could be reasonably assumed to be fixed. Pozzi and Schivardi (2016) take advantage that estimates of the elasticity of demand could be obtained from self-reported information by firms. Another popular method to disentangle productivity and demand shocks is to impose demand structure, as in the recent paper by Kumar and Zhang (2019), for example. Jaumandreu and Yin (2019) impose less demand side structure and allows for the flexible dynamics of latent productivity and demand advantages while relying on firms that sell in both a domestic and export market for identification. Rubens (2021) develops a model of firms with heterogeneous productivity that face imperfect competition in both input and output markets. The estimation procedure presented relies on a linear dynamic process of latent productivity.

I propose to jointly estimate the production function, demand function, labor

[5] A paper with a related empirical focus is Carlsson et al. (2020) which estimates the effects of firm-level productivity shocks and demand shocks on labor flows using a VAR framework.

supply function, and the processes for productivity and demand advantages using nonlinear dynamic panel data methods that allow for latent variables. The proposed model extends standard models, such as that in Jaumandreu and Yin (2019) and Rubens (2021), to allow for labor market power and nonlinear dynamics in productivity and demand advantages. This empirical framework leverages the novel results in the identification and estimation of these models (Hu and Schennach, 2008; Arellano and Bonhomme, 2016, 2017; Arellano et al., 2017). The identification and estimation differ from currently popular methods of production function estimation that center around the so-called proxy method or control function approaches and its extensions (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg et al., 2015; Gandhi et al., 2020). It also extends results on the estimation of production functions using linear dynamic panel data models (Arellano and Bond, 1991; Blundell and Bond, 1998, 2000; Ackerberg, 2020). I highlight three main advantages of the proposed method over the existing literature. First, we can allow for multidimensional unobserved firm heterogeneity which many of the control function approaches have difficulty with. Second, we allow for nonlinear dynamics in the latent processes of productivity and demand advantages, which cannot be accommodated in linear dynamics panel methods. Lastly, we also flexibly estimate empirical policy functions for input decisions and wage-setting which will allow us to study firm responses to shocks.

Lastly, this paper contributes to the small but growing literature that quantifies responses of economic agents to latent shocks using flexible nonlinear semi-structural empirical frameworks. The recent novel results on the identification and estimation of nonlinear dynamic panel data models with latent variables (see, Hu and Schennach, 2008; Arellano and Bonhomme, 2016, 2017) has provided us a set of practical tools to flexibly measure nonlinearities in the responses of economic agents to shocks by estimating empirical response functions. The main advantage of directly estimating the decision rules of the agents as opposed to specifying a structural model is that the estimates of the marginal quantities are robust to misspecification of the model primitives. In this paper, I focus on the estimation of the wage pass-through which is a marginal quantity as it is the derivative of the wage-setting policy function with respect to productivity and demand shocks.

A couple of papers have measured marginal quantities in various contexts using this type of flexible framework. Arellano et al. (2017) study nonlinear income dynamics and the consumption responses of households to shocks in the persistent and transitory components of earnings, extending the framework by Blundell et al. (2008) who estimate linear covariance structures. They find that the persistence of earnings varies by the size and sign of the current shock and that these nonlinear dynamics drives heterogeneous consumption responses to earnings shocks. Gálvez (2017) studies the role of nonlinear household income risk on portfolio choices with participation costs. Similarly, past earnings histories and the size and durability of current income shocks are important to explain heterogeneous extensive and intensive margin responses in portfolio choice. Without the aid of a structural model, further policy counterfactuals cannot be performed.

In concurrent work, Aguirre et al. (2021) estimate a semi-structural model to study firm investment decisions in the presence of borrowing constraints. Similarly, I also study firm responses however I examine responses to both productivity and demand shocks. As such, my framework differs from theirs in that I allow for imperfect competition in both output and input markets. The final objects of interest also differ. I am interested in the wage pass-through of firm shocks.

Outline of the paper. The remainder of this paper proceeds as follows. In Section 2, I describe the merged data sets used in the analysis, and provide a brief background on wage-setting in Portugal. In Section 3, I introduce an empirical framework that distinguishes productivity from demand heterogeneity among firms who have labor market power. I also discuss identification and estimation, as well as present results on model fit. In Section 5, I document facts on the cross-sectional heterogeneity and dynamics of productivity and demand heterogeneity. In Section 6, I discuss the dynamics of wages as it relates to adjustment costs and the pass-through of productivity and demand shocks. Finally, I conclude in Section 7 with a summary of the findings and discussion of future directions of work. An Appendix contains additional results and technical details.

2 Data and institutional setting

2.1 Data description and variable construction

I combine a matched employer-employee data set, firm-level balance sheet and income statement data set, and a manufacturing survey to form a longitudinal dataset of firms and workers. I focus on the manufacturing sector in continental Portugal over the period 2012–2018.

Portugal's matched employer-employee data set is the Quadros de Pessoal (QP) which covers the universe of firms.[6] It draws from a compulsory annual census of firms that employ at least one worker conducted by the Portuguese Ministry of Labor and Social Security.[7] Worker information is available for individuals working in the firm in a reference week of the month of October of each year. Firms and workers can be tracked over time and thus, the QP also forms a panel data set for both workers and firms. Worker information in the QP is detailed and includes demographics (age, gender, education), earnings (base wage, overtime pay, regularly paid supplements, and irregular supplements), hours (normal hours and overtime hours), occupation, and position in the firm hierarchy.

Balance sheet and income statement information is obtained from Sistema de Contas Integradas das Empresas (SCIE). This information on business accounts is available for all firms excluding public firms, not-for-profits, and financial firms. This is collected as a compulsory survey to firms by the Portuguese Tax Authority. On the other hand, information at the firm-product-year level is obtained from the Inquérito Anual à Produção (IAPI) which is a manufacturing survey conducted on a sample of firms with at least 20 employees. As it is only available for a subset of large manufacturing firms, this data set limits the scope of the paper. The caveat then is that the results may only be representative of manufacturing firms with more than 20 employees. Total production recorded in the IAPI accounts for 90% of total value of production. For each firm-product, I observe the volume of the product sold (measured in specific units i.e., liters, kilograms, pieces, etc.) and the value of the products sold in euros. From which, average selling price per unit of the product can be computed.

Multi-product firms, real output, and price indices. The multi-product nature of most firms in the manufacturing sector is an important feature that needs to be

[6] This data set has been used in multiple studies of both firms and/or workers. To name a couple: Cardoso and Portela (2009) uses QP to study wage pass-through of sales using linear covariance structures as in Guiso et al. (2005). Caliendo et al. (2020) combines QP with balance sheet information and a manufacturing survey (as in this paper) to study the productivity effects of hierarchical reorganization.

[7] Firms are required to make this information publicly available to its workers in the workplace. This helps in monitoring the accuracy and improving the reliability of the information provided.

accounted for in the model. However, as it is not the main focus of this paper, I adopt a parsimonious solution. In general terms, I compute the real output of a firm by valuing the quantities produced using fixed prices that I set to be constant across firms as well as over time (chosen as the median price of the product in 2018). Prices are then an index obtained from dividing total revenues, or nominal output, by real output. In Appendix A, I present the details and provide an intuitive demonstration of how this approach separates quantity and price variation in the presence of multiple products which preserves not only time-series variation but cross-sectional variation, as well.

Sample selection on workers and firms. Though data on all manufacturing sectors are available, in this paper, I focus on firms in three specific manufacturing sectors: (i) food, (ii) clothing, and (iii) metal products, which are sectors 10, 14, and 25 based on the 2-digit Portuguese Classification of Economic Activity (Classificação Portuguesa das Actividades Económicas, CAE Rev. 3), respectively.[8] This is solely to alleviate the computational burden. These sectors are the largest in terms of the number of firms and cover about 39% of total manufacturing employment and 24% of total manufacturing sales over 2012–2018. The identification of the model, discussed in Section 4.2, relies on the panel dimension of the data. Motivated by this, I restrict the data to include only firms that I observe for at least 2 consecutive years.

The matched employee-employer data is important for us to control for worker heterogeneity. As discussed later, I compute a firm-level average wage rate net of worker characteristics. For this exercise, I restrict the analysis to workers aged 18 to 55. The wage concept I use includes wages paid for both normal and supplementary hours worked.

2.2 Wage-setting in Portugal

Collective bargaining agreements, often covering entire industries, are prevalent in the Portuguese economy. In practice, their coverage extends to workers not affiliated with unions. However, these collective agreements are partially offset by firm-specific agreements which allows firms room to maneuver when setting wages

[8] CAE Rev. 3 closely maps to NACE codes.

(Bover et al., 2000; Cardoso and Portugal, 2005; Card and Cardoso, 2021). Specifically, two main features of the Portuguese economy admit this room to maneuver: (i) collective bargaining agreements have narrow coverage and do not restrict links between wage dynamics and firm performance, and (ii) prevailing wages are well-above the wage floors set in collective bargaining agreements.

As many collective bargaining agreements cover a large number of firms in a wide range of economic circumstances, the content of collective bargaining agreements necessarily have limited jurisdiction and primarily focus on setting minimum working conditions such as the minimum monthly base wage of different categories of workers, overtime pay, and normal duration of work (Cardoso and Portugal, 2005). Firms are, thus, free to adjust wages based on firm-specific conditions as long as they satisfy the negotiated wage floors. Moreover, the frequency of wage changes is not covered by the collective agreements.

The flexibility of changing wages (downward adjustment in particular) is most relevant if prevailing wages are above the wage floors set in the collective bargaining agreements. Card and Cardoso (2021) identify the collective bargaining agreement relevant to individuals workers in the Portuguese matched employer-employee data set and show that a typical worker receives a premium of around 20% over their relevant negotiated wage floor.[9] This shows that firms adjust the wages they pay on firm-specific conditions.

3 A semi-structural model of firm production with imperfectly competitive product and labor markets

We do not directly observe productivity, consumer preferences, or worker preferences and so measuring their pass-through to wages is not straightforward. Moreover, firm decisions are endogenous to these unobserved objects. We, however, have access to imperfect measures of these objects and observe choices that were based on these. It is then important to use the lens of a model to disentangle productivity from demand advantages from labor market advantages in the data and study how they affect decisions of firms.

As a measurement exercise, it is crucial that we impose limited structure on the

[9]The presence of wage rates well-above negotiated wage floors is similar to other countries with systems of sectoral bargaining, for instance Spain and Italy (Adamopoulou and Villanueva, 2022).

data to back out the real relationships between the observed data and the unobserved objects. In the model discussed below, I present an attempt to accommodate flexibility along multiple dimensions. First, I model these unobserved components as general first-order Markov processes. In the context of productivity, this is in contrast to empirical models of firm dynamics in the macroeconomics literature where productivity is often parametrized to follow an $AR(1)$ process. On the other hand, this is a similar flexible assumption maintained in production function estimation using control function approaches (e.g., Olley and Pakes, 1996). This is important because there are plausible scenarios that suggest rich dynamics in productivity, demand, and labor market advantages.

Second, I model the responses of the firms to idiosyncratic shocks flexibly in that some structural objects need not be specified. In the study of wage pass-through it is important, for instance, to accommodate wage adjustment costs. Thus, in the model, I allow wages to depend on past wages in an arbitrary manner—that is, without specifying the exact shape of these adjustment costs. The main virtue of this is that our measurement is not tied to certain parametrizations or modeling choices of structural objects. However, we are limited by the types of counterfactual analyses that could be credibly performed as in the Lucas (1976) critique.

In what follows, I will describe this flexible empirical framework for firms and workers that distinguishes productivity and demand shocks in an environment that allows for imperfect competition in the output and input markets.

Preliminaries. Time is indexed by t and is discrete with time points representing years. Firms are indexed by j and workers are indexed by i. The specific firm that a worker i works for in time t is $j(i,t)$. Firms differ along three main dimensions: productivity, demand advantages and labor market advantages. They operate and compete with other firms in their relevant local product and labor market which, in this paper, will be at the industry level determined by the 2-digit industry codes.

Production Technology. Firms use fixed tangible capital (K_{jt}), labor (L_{jt}), and intermediate goods (M_{jt}) to produce a homogeneous good (whose quantity is represented by Y_{jt}).[10] Firms are heterogeneous in their efficiency to aggregate inputs

[10]The assumption of physically homogeneous final goods is an abstraction to distinguish technological efficiency from product market heterogeneity. This assumption is more plausible for some products and industries than others. Foster et al. (2008), for instance, focus their analysis of the

captured by a factor-neutral productivity term $\exp(\omega_{jt})$.[11] The production function of the firm is given by

$$Y_{jt} = \exp(\omega_{jt}) \times Q^Y(K_{jt}, L_{jt}, M_{jt}) \times \epsilon^Y_{jt} \tag{1}$$

where Q^Y is a function common to all firms that describes how inputs are aggregated, and ϵ^Y_{jt} are idiosyncratic, serially uncorrelated shocks to production.[12] The productivity term $\exp(\omega_{jt})$ is observable to the firm at the beginning of the period but ϵ^Y_{jt} is not. Labor, measured by the number of employed workers, is treated as homogeneous. This framework can be extended to include the treatment of heterogeneous labor types by allowing heterogeneous labor inputs to separately enter the production function (e.g. high-education vs low-education workers).[13] It is important to note that since inputs are endogenously chosen with productivity being a key state variable, this production function cannot be estimated just by OLS. As such, we will have to fully specify the empirical policy functions of the inputs given productivity to complete the model.

Operating environment: product and labor markets. Firms operate in an environment with imperfect competition in both product (output) and labor (input) markets.

In the product market, firms compete in prices under *monopolistic competition*. This particular market structure is plausible considering there are a large number of firms within each manufacturing sector such that strategic interaction is difficult. The residual demand curve for the final good faced by individual firms relates

roles of productivity and demand advantages on firm entry and exit to firms that produce one of eleven specific manufacturing products they consider quasi-homogeneous.

[11] One can also allow for the interaction of the scalar unobservable with the inputs. In such a case, input elasticities differ across firms because of this unobserved factor. Alternatively, one can also interpret it as heterogeneity in capital- or labor-augmenting productivity driven by a single factor.

[12] Without additional information, we cannot separately identify transitory shocks to productivity from classical measurement error.

[13] Alternatively, one can allow for a labor-augmenting productivity term to capture between-firm differences in the productivity composition of the worker pool (Doraszelski and Jaumandreu, 2018; Demirer, 2019). This introduces more nuanced identification issues and its study is of interest for further research. Chan et al. (2019) take another approach and adjust the labor input for efficiency based on unobserved worker "ability" taking advantage of the panel dimension in matched employer-employee data to estimate a worker fixed effect.

quantity demanded (Q_{jt}^{dd}) and prices (P_{jt}) in the following manner:

$$Q_{jt}^{dd} = Q^D(P_{jt}, \delta_{jt}) \times \epsilon_{jt}^D \tag{2}$$

where δ_{jt} are demand advantages that capture differences in quantities demanded between firms selling at the same price. These demand advantages capture horizontal differentiation in the final output of firms, primarily resulting from idiosyncratic consumer preferences. There are multiple fixed or exogenous sources of horizontal differentiation between firms' output. Firstly, the most commonly discussed example of horizontal differentiation is heterogeneous preferences of consumers on product characteristics; for instance, local/foreign makes of automobiles (Goldberg, 1995), or sugar content in ready-to-eat cereals (Nevo, 2001). Secondly, horizontal demand advantages may arise from spatial differentiation as in the Hotelling (1929) model of the linear street. The general argument says that with transportation costs, demand of buyers will be divided among producers accounting for proximity. Thus, with a non-uniform distribution of buyers across space, some firms will have better demand advantages simply because they are located closer to the mass of consumers. Miller and Osborne (2014), for example, study spatial differentiation and price discrimination in the cement industry in the US. Thirdly, demand advantages may also arise from complex and idiosyncratic historical reasons. Bronnenberg et al. (2007) document that by the end of the twentieth century, most consumer goods were dominated by a small number of brands in terms of value of sales. Taking advantage of spatial variation in the dominant brands, Bronnenberg et al. (2009) show that this dominance does not reflect quality but can be tied to order of entry (which in some cases were over more than a century ago).[14] Moreover, they show high persistence of brand dominance.

On the other hand, the market structure of the labor market is *monopsonistic competition*. Firms set wages (w_{jt}) facing idiosyncratic residual labor supply curves given by

$$L_{jt} = Q^L(w_{jt}, S_{jt}) \times \epsilon_{jt}^L \tag{3}$$

[14] As a case study of serendipity being a source of demand advantages, we can look to the rise to popularity of Piaggio's Vespa motor scooter in the 1950's. The 1953 debut film of Audrey Hepburn entitled *Roman Holiday* featured the Vespa for a few minutes. Reports suggest that 100,000 units were sold right after the release of the movie when Piaggio was initially only planning for a total run of 2,000 units (Sonnadara, 2013; de Burton, 2021). Vespa's fall in popularity in the US is arguably also by stroke of luck. In 1983, imports of the Vespa stopped after the US tightened federal laws on emissions which Piaggio could not adapt to (Sonnadara, 2013).

which is a function of set firm-level wages w_{jt} and labor market advantages S_{jt}. Such a labor supply curve could be microfounded using a model of wage-posting where firms are characterized by a set of amenities which workers value differentially (Card et al., 2018). As in the model for amenities, these labor market advantages capture job-specific and location-specific non-pecuniary compensation or benefits which has been shown to be substantially different even among firms within the same market (Sorkin, 2018; Lamadon et al., 2019; Sockin, 2022).[15] However, labor market advantages may also reflect a firm's access to larger local labor markets or more streamlined hiring practices.

It is again important to note that prices and wages are endogenous in the model and are choices that depend on the unobserved demand and labor market advantages. As such, we would need to complete the model by specifying the empirical policy functions. In the working model I will build, prices and materials are static choices that are made simultaneously. As such, once materials are chosen, prices are also set. Thus, we only need to specify the materials policy function in the statistical model.

Dynamics of firm heterogeneity. As described above, firms are heterogeneous along three dimensions: productivity, demand, and labor market advantages. These firm advantages are persistent and evolve stochastically according to exogenous first-order Markov processes.[16] In particular,

$$\omega_{jt} = \widetilde{Q}^{\omega}(\omega_{jt-1}, v_{jt}^{\omega}) \tag{4}$$

$$\delta_{jt} = \widetilde{Q}^{\delta}(\delta_{jt-1}, v_{jt}^{\delta}) \tag{5}$$

$$S_{jt} = \widetilde{Q}^{S}(S_{jt-1}, v_{jt}^{S}) \tag{6}$$

[15] In the literature, amenities are typically modeled as fixed characteristics of the firm. There are scenarios for which we might think that these could plausibly evolve exogenously over time. For instance, a new train station may open near the firm which makes it easier for workers to commute to work.

[16] Here we do not allow dynamics to depend on the business cycle, or calendar time, in general. The results by Salgado et al. (2020) would suggest there may be interesting business cycle patterns in the distribution of shocks to firm outcomes that we are not able to capture by this simplified model. In a preliminary processing step, I take out sector-time averages of the variables of interest which controls for the level differences that may be brought by the business cycle. However, the choice to forgo modeling distributional dependence on calendar time is motivated by the short time period of interest, 2012-2018, when the Portuguese macroeconomy appears to be stable. Extensions of the model to include aggregate shocks is subject of future research.

where $\widetilde{Q}(\cdot, \tau)$ are univariate conditional quantile functions for $\tau \in (0,1)$, and $v^{\omega}_{jt}, v^{\delta}_{jt}, v^{S}_{jt}$ are idiosyncratic shocks to productivity, demand, and labor market advantages, respectively. These idiosyncratic shocks have uniform marginals and may be contemporaneously correlated, but the vector $(v^{\omega}_{jt}, v^{\delta}_{jt}, v^{S}_{jt})$ is serially uncorrelated. Correlation in productivity, demand, and labor market advantages may come from correlation in initial values or through correlated shocks. I restrict the model such that there are no cross-effects, for instance with past demand advantages directly affecting current productivity, but this extension can be accommodated.

The flexible specification in the dynamics of these latent objects allows for us to study rich dynamics which may have implications on firm decisions such as capital investment, research and development, advertising, or wage-setting. For instance, these general Markov models allow the persistence of productivity, demand, and labor market advantages to differ depending on its level or the size and sign of the shock. To illustrate in more concrete terms, a firm with low productivity and bad position in the output market might be lucky to hire a new manager that overhauls the production process and creates a better brand position for the product. This shock to the firm will likely boost both productivity and demand advantages, and its history as an unproductive firm with no brand value becomes irrelevant. In such a case, the persistence of the past productivity and demand advantage is lower with a large enough positive shock. Moreover, this scenario also illustrates that shocks to productivity and demand advantages may be correlated.

Wage-setting. Faced with their individual residual labor supply curves described in Equation 3, firms set firm-level wages to determine the amount of labor hired for production. We also consider an evironment where there may be wage adjustment costs. The empirical wage-setting equation of the firm is given by

$$w_{jt} = Q^{w}(w_{jt-1}, K_{jt}, \omega_{jt}, \delta_{jt}, S_{jt}) \times \epsilon^{w}_{jt}. \tag{7}$$

Without adjustment costs, firms set wages as a markdown to marginal revenue product of labor (MRPL). These markdowns are a function of the labor supply elasticities. Both the MRPL and the labor elasticity are jointly determined by the state variables $(K_{jt}, \omega_{jt}, \delta_{jt}, S_{jt})$. However, in the presence of wage adjustment costs, firms set wages taking into account the associated adjustment cost and the implications for future wage-setting. In such case, wages will also be a function of past wages w_{jt-1}. Note that this wage equation is a reduced-form policy function which we will estimate from the data. It is, for instance, consistent with various parametrizations of the wage adjustment costs which we do not need to specify. The idiosyncratic serially uncorrelated shock ϵ^w_{jt} may represent optimization errors, for example.

Only the firm-level wage rate, w_{jt} is relevant to determine the labor quantity supplied by the market. However, there is observed heterogeneity in wages across workers hired by the same firm. To account for this, individual wages are modeled to have a component that captures exogenous returns to time-varying characteristics x_{it} (e.g., age, tenure, occupation) and a fixed worker component α_i; that is,

$$w_{it} = w_{j(i,t),t} \times Q^{HC}(x_{it}, \alpha_i) \times \epsilon^w_{it} \tag{8}$$

where Q^{HC} is the function that summarizes the market-determined returns to observed characteristics. This proportionality relationship between the firm-level wage and an individual level wage determined in the market is similar to the wage determination in Heckman and Sedlacek (1985).[17] Separability of the firm-specific component from the worker component is convenient for estimation as the human capital component can be estimated using wages in the matched employer-employee data alone.

Price setting and materials input. Firms make two additional static decisions: output prices and materials input. These are decisions made simultaneously after wages are set and labor is realized. Prices are not subject to adjustment costs, which is plausible considering the annual frequency of the data. Moreover, results by Marques et al. (2010) suggest that prices are adjusted more often than wages.

Specifically, given the firm's individual demand function and its cost function (implied by its production technology), firms choose prices to maximize profits equating marginal revenue to marginal costs. The reduced-form empirical pricing function of the firm is given by

$$P_{jt} = Q^P(K_{jt}, L_{jt}, \omega_{jt}, \delta_{jt}, S_{jt}) \times \epsilon^P_{jt}. \tag{9}$$

[17] Non-proportional wages may be accommodated, for example, by specifying multiple wage functions depending on worker characteristics, such as education.

Intuitively, price-setting is a function of the demand faced by the firm (characterized by its demand advantages δ_{jt}), and the firm's marginal cost which is a function of level of production (K_{jt}, L_{jt}), wages (w_{jt}), productivity ω_{jt}, and labor market advantages S_j. Equivalently, we can instead specify the material input equation:

$$M_{jt} = Q^M(K_{jt}, L_{jt}, \omega_{jt}, \delta_{jt}, S_{jt}) \times \epsilon^M_{jt} \tag{10}$$

where Q^M is the function that describes a firm's material demand as a function of state variables, and ϵ^M_{jt} is an idiosyncratic, serially uncorrelated shock in the firm's material usage. The idiosyncratic shock captures unexpected inefficiencies (waste) or savings in the usage of materials, or can represent random optimization errors.

There are two things to note about these equations. First, given the setting, once material input is chosen, then prices are simultaneously determined—this implies that the price-setting equation is redundant once we have specified the material input policy function. Second, the decision for materials depend on a number of unobservables including both productivity and demand advantages. As such, standard control function approaches that use materials as a proxy variable (e.g., Levinsohn and Petrin, 2003; Ackerberg et al., 2015) cannot be applied as productivity cannot be fully inverted from observing materials, capital and labor alone.

Capital and investment. To complete the model, we specify a law-of-motion for the predetermined input, capital:

$$K_{jt} = Q^K(K_{jt-1}, w_{jt-1}, \omega_{jt-1}, \delta_{jt-1}, S_{jt-1}) \times \epsilon^K_{jt}. \tag{11}$$

The function Q^K tells us how last period capital K_{jt-1} is depreciated and how previous investment decisions affect current capital. The idiosyncratic shock ϵ^K_{jt} captures possible shocks to fixed capital (i.e., unexpected savings or depreciation) or measurement error. This reduced-form law-of-motion of capital subsumes the dynamic investment decision of the firm which is a function of the previous level of capital (possibly, as a state variable to determine capital adjustment costs), as well as additional determinants of capital investment including the previous levels of wages, productivity, demand, and labor market advantages.

4 Parametrization, identification, and estimation

In this section, I will specify a flexible parametric version of the model above. The specifications for the dynamic processes of the latent firm heterogeneity will be as in Arellano et al. (2017) where we flexibly specify the conditional quantile functions of the processes. For the policy functions, I specify tighter specifications where I consider flexible dependence of the conditional mean on the state variables. I then proceed with a discussion of parametric and non-parametric identification, and provide a simulation-based algorithm for estimation.

4.1 Empirical specification

Production, output demand, and labor supply. The general model is flexible enough to allow for arbitrary specifications of the production function, output demand function, and labor supply equation—Equations (1), (2), and (3), respectively. In this paper, I consider more parsimonious specifications although, in principle, more general formulations are possible.

I consider simpler specifications for the production function and residual output demand functions. In particular, I consider a production function that is log-linear in the inputs and productivity, i.e., Cobb-Douglas:

$$y_{jt} = \beta_k k_{jt} + \beta_m m_{jt} + \beta_\ell \ell_{jt} + \omega_{jt} + \varepsilon^y_{jt}. \tag{12}$$

As a simplification, we specify $\varepsilon^y_{jt} \sim \mathcal{N}(0, \sigma^2_y)$.[18] Similarly, the output demand function is log-linear in prices and demand advantages:

$$y_{jt} = \alpha_p p_{jt} + \delta_{jt} + \varepsilon^d_{jt}, \tag{13}$$

where y_{jt} is log output, p_{jt} is log prices, and δ_{jt} is the demand advantage. Then, α_p captures the elasticity of demand faced by the firm. For simplicity, I assume that $\varepsilon^d_{jt} \sim \mathcal{N}(0, \sigma^2_d)$. This specification mimics the demand model in Jaumandreu and Yin (2019) who interpret this model as a first-order approximation to arbitrary demand functions.

As the focus of this paper is on the labor market, I consider a more flexible

[18] It is straightforward to allow for an arbitrary distribution of the idiosyncratic innovation to production by instead specifying the conditional quantile functions of ε^y_{jt}.

specfication of the labor supply function to allow for heterogeneity in labor market power across firms. In particular,

$$\ell_{jt} = \theta_w \ln w_{jt} + \theta_{w2}(\ln w_{jt})^2 + \theta_{ws} \ln w_{jt} \times S_{jt} + \theta_{w2s}(\ln w_{jt})^2 \times S_{jt} + S_{jt} + \varepsilon^{\ell}_{jt}, \quad (14)$$

where $\varepsilon^{\ell}_{jt} \sim \mathcal{N}(0, \sigma^2_{\ell})$. The elasticity of labor supply with respect to wages is $(\theta_w + 2\theta_{w2} \ln w_{jt} + S_{jt} + 2\theta_{w2s} \ln w_{jt} \times S_{jt})$ and thus varies across firms depending on the level of wages and demand advantages.

As a normalization, without loss of generality, I do not include a constant in the production function, product demand equation, or labor supply function.

Dynamics of latent firm heterogeneity. As previously discussed in Section 3, the joint dynamics of the latent firm heterogeneity (productivity, demand, and labor market advantages) is fully specified by Equations (4), (5), and (6). Instead of specifying a copula model that would describe the dependence structure of $(v^{\omega}_{jt}, v^{\delta}_{jt}, v^{S}_{jt})$, I consider the following triangular formulation:

$$\omega_{jt} = Q^{\omega}(w_{jt-1}, u^{\omega}_{jt}) \quad (15)$$

$$\delta_{jt} = Q^{\delta}(\delta_{jt-1}, u^{\omega}_{jt}, u^{\delta}_{jt}) \quad (16)$$

$$S_{jt} = Q^{S}(S_{jt-1}, u^{\omega}_{jt}, u^{\delta}_{jt}, u^{S}_{jt}) \quad (17)$$

where $u^{\omega}_{jt}, u^{\delta}_{jt}, u^{S}_{jt}$ are uniformly distributed, mutually independent, and serially uncorrelated. Intuitively, this could be thought of as an othogonalization of the shocks in the univariate processes. In Appendix B, I discuss in more detail how this triangular representation relates to the system of univariate dynamic processes in Equations (4), (5), and (6).

These conditional quantiles are parametrized in the following way, extending Arellano et al. (2017):

$$Q^{\omega}(\omega_{jt-1}, \tau) = \sum_{k=0}^{K} a^{\omega}_k(\tau)\varphi^{\omega}_k(\omega_{jt-1}), \quad (18)$$

$$Q^{\delta}(\delta_{jt-1}, u^{\omega}_{jt}, \tau) = \sum_{k=0}^{K} a^{\delta}_k(\tau)\varphi^{\delta}_k(\delta_{jt-1}, \Phi^{-1}(u^{\omega}_{jt})), \quad (19)$$

$$Q^{S}(S_{jt-1}, u^{\omega}_{jt}, u^{\delta}_{jt}, \tau) = \sum_{k=0}^{K} a^{S}_k(\tau)\varphi^{S}_k(S_{jt-1}, \Phi^{-1}(u^{\omega}_{jt}), \Phi^{-1}(u^{\delta}_{jt})), \quad (20)$$

for $t = 2, ..., T$, where φ_k^{ω}, φ_k^{δ}, and φ_k^{S} are dictionaries of basis functions for $k = 0, 1, ...$, and $\Phi^{-1}(\cdot)$ is the inverse of the standard normal cumulative distribution function. As in Arellano et al. (2017), I choose these basis functions to be low-order products of Hermite polynomials with $\varphi_0^{\omega} = \varphi_0^{\delta} = \varphi_0^{S} = 0$.

Note that the conditional quantiles above involve a continuum of parameters indexed by $\tau \in (0, 1)$. In practice, I borrow the insight of Arellano and Bonhomme (2016) and approximate the coefficients of the conditional quantile functions. Consider Equation (18) to illustrate. We approximate the coefficients $a_k^{\omega}(\tau)$ as piecewise-linear interpolating splines on a grid $[\tau_1, \tau_2]$, $[\tau_2, \tau_3]$, ..., $[\tau_{L-1}, \tau_L]$ contained in the unit interval. I consider an equidistant grid so that $\tau_\ell = \ell/(L+1)$, and choose $L = 11$. To extend this to the tails, we specify that the intercept $a_0^{\omega}(\tau)$ match the quantiles of an exponential distribution on $(0, \tau_1]$ and $[\tau_L, 1)$ with exponential parameters λ_-^{ω} and λ_+^{ω}, respectively. Similar approximations are used for the coefficients in Equations (19) and (20).

To complete the model for the dynamics of the latent heterogeneity, we need to specify the initial distributions of ω_{j1}, δ_{j1}, and S_{j1}. As the dynamics of the three latent firm components interact nonlinearly, it is likely that the initial conditions also depend on each other in nonlinear ways. Taking a flexible random effects approach, I allow the distribution of the initial values to depend on the age of the firm at first observation age_{j1} and log capital at initial observation k_{j1}. Again, we consider a triangular structure to flexibly model the dependence of the initial conditions:

$$S_{j1} = \sum_{k=0}^{K} b_k^{S_1} \varphi_k^{S_1}(k_{j1}, \text{age}_{j1}) + \varepsilon_{j1}^{S_1}, \tag{21}$$

$$\delta_{j1} = \sum_{k=0}^{K} b_k^{\delta_1} \varphi_k^{\delta_1}(S_{j1}, k_{j1}, \text{age}_{j1}) + \varepsilon_{j1}^{\delta_1}, \tag{22}$$

$$\omega_{j1} = \sum_{k=0}^{K} b_k^{\omega_1} \varphi_k^{\omega_1}(\delta_{j1}, S_{j1}, k_{j1}, \text{age}_{j1}) + \varepsilon_{j1}^{\omega_1}, \tag{23}$$

for a given set of basis functions $\varphi_k^{S_1}$, $\varphi_k^{\delta_1}$, and $\varphi_k^{\omega_1}$ chosen to be low-order products of Hermite polynomials. I specify that $\varepsilon_{j1}^{S_1} \sim \mathcal{N}(0, \sigma_{S_1}^2)$, $\varepsilon_{j1}^{\delta_1} \sim \mathcal{N}(0, \sigma_{\delta_1}^2)$, and $\varepsilon_{j1}^{\omega_1} \sim \mathcal{N}(0, \sigma_{\omega_1}^2)$. We can alternatively specify the full conditional distributions of the initial conditions by specifying the conditional quantiles, as we did for the dynamics. I opt to use a more parsimonious specification and only specify the flexible dependence through the conditional mean.[19]

Individual wages. We assume the following specification for the log of individual wages:

$$\ln w_{it} = \ln w_{j(i,t),t} + \underbrace{\beta_1 \text{age}_{it} + \beta_2 \text{age}_{it}^2 + \alpha_t + \alpha_i}_{\ln Q^{HC}(x_{it},\alpha_i)} + \varepsilon_{it}^{w}, \tag{24}$$

where α_t and α_i are year and individual fixed effects, respectively. Under the strong assumption that workers do not sort into firms by individual fixed effects, then we can estimate the human capital part of the wage equation in a first step. In the framework presented previously, we treat all workers in a similar way in production (i.e., perfectly substitutable between each other), then it is consistent to assume that there is no sorting. I acknowledge that this is a strong, counterfactual assumption and future work will attempt to account for this.[20]

Empirical response functions. We now specify the conditional distributions of wages, materials, and capital based on the reduced-form policy functions of the firm in Equations (7), (10), and (11), respectively. I consider flexible specifications of the conditional mean:

$$\ln w_{jt} = \sum_{k=0}^{K} b_k^w \varphi_k^w(\ln w_{jt-1}, k_{jt}, \omega_{jt}, \delta_{jt}, S_{jt}) + \varepsilon_{jt}^w \text{ for } t = 2, ..., T, \tag{25}$$

$$m_{jt} = \sum_{k=0}^{K} b_k^m \varphi_k^m(k_{jt}, \ell_{jt}, \omega_{jt}, \delta_{jt}, S_{jt}) + \varepsilon_{jt}^m \text{ for } t = 1, ..., T, \tag{26}$$

$$k_{jt} = \sum_{k=0}^{K} b_k^k \varphi_k^k(k_{jt-1}, w_{jt-1}, \omega_{jt-1}, \delta_{jt-1}, S_{jt-1}) + \varepsilon_{jt}^k \text{ for } t = 2, ..., T, \tag{27}$$

[19] This is equivalent to modeling the conditional quantile function as

$$Q^d(\mathbf{v}_{jt}^z, \tau) = \sum_{k=1}^{K} b_k^z \varphi_k^z(\mathbf{v}_{jt}^z) + b_0^z(\tau),$$

and choosing $b_0^z(\tau) = b^z + \sigma_z \Phi^{-1}(\tau)$. This model differs from Equations (18), (19) and (20) in that only the intercept depends on τ in a manner implied by the Normality assumption.

[20] As mentioned previously, we can classify workers into types (either by observables or unobservables) and model firm-level wages for each type of worker. Moreover, we would include different labor types in the production function.

for basis functions φ_k^w, φ_k^m, and φ_k^k. Moreover, I choose $\varepsilon_{jt}^w \sim \mathcal{N}(0, \sigma_w^2)$, $\varepsilon_{jt}^m \sim \mathcal{N}(0, \sigma_m^2)$, and $\varepsilon_{jt}^k \sim \mathcal{N}(0, \sigma_k^2)$. To complete the model, we need to further specify the initial distribution of wages conditional on capital and age at first observation, as well as productivity, demand, and labor market advantages at first observation:

$$\ln w_{j1} = \sum_{k=0}^{K} b_k^{w_1} \varphi_k^{w_1}(k_{j1}, \text{age}_{j1}, \omega_{j1}, \delta_{j1}, S_{j1}) + \varepsilon_{j1}^{w_1}, \tag{28}$$

where $\varphi_k^{w_1}$ are low-order products of Hermite polynomials, and $\varepsilon_{j1}^{w_1} \sim \mathcal{N}(0, \sigma_{w_1}^2)$.

4.2 Identification

Identification is challenging because productivity, demand, and labor market advantages are unobserved to the econometrician. Moreover, firm decisions are endogenous to these unobservable components. The Markovian assumptions for the states together with the fact that the production function, demand function, labor supply function, and policy functions are static responses are the key building blocks for identification. These assumptions allow us to implicitly use variables from other periods as instrumental variables. The static nature of the response functions justify the exclusion restrictions while the Markovian assumptions are linked to instrument relevance.

In Appendix C, I discuss in detail identification in a simple linearized version of the model. We use past outputs and choices as instruments in quasi-first-differenced versions of the model. For instance, after differencing out the role of past productivity in the production function, we can use past inputs as proxies for demand shocks as instruments to identify the production function parameters. Similarly, after appropriate transformations, we can use past prices and inputs as instruments, proxying productivity shocks, to identify the demand elasticity. The more general parametric model is identified under less tighter conditions.

Nonparametric identification would be important so that we are not dependent on parametric assumptions. Nonparametric identification is plausible extending arguments from Hu and Schennach (2008), Hu and Shum (2012), Arellano and Bonhomme (2017) and Arellano et al. (2017) who establish conditions under which dynamic nonlinear panel data models with latent variables are identified.[21]

[21]Nonparametric identification of the production function in a flexible semi-structural framework

4.3 Estimation

The model is estimated separately by 2-digit sector. Results presented in the paper are aggregated, weighted by sales, but some results of individual sectors are in the appendix. In practice, there are likely aggregate effects which we are not able to capture in the model. Thus, I work with residuals of all the variables of interest after partialling-out sector-year fixed effects in a preliminary processing step.

Estimating all the parameters of the model jointly is challenging. As such, I follow a two-stage estimation procedure where the demand elasticity in Equation (13) is first estimated in a preliminary stage. The first-stage estimation is based on a generalized method of moments (GMM) estimation allowing for the nonlinear dynamics in the demand advantage process. I provide a more detailed discussion in Appendix D.

The estimated demand elasticity is taken as given in the second stage where the rest of the parameters are estimated. Following Arellano and Bonhomme (2016) and Arellano et al. (2017), the second stage is based on a stochastic EM algorithm. This algorithm alternates between two steps until convergence. In the first step, latent variables (i.e., productivity, demand, and labor market advantages) are drawn from their posterior distribution using Markov-Chain Monte Carlo (MCMC) techniques. In the second step, the parameters of the model are updated taking as given the latent draws. The second step is a series of linear quantile regressions, least squares regressions, and nonlinear regressions. Alternating these two steps produces a Markov chain of draws of the parameter estimates. The final parameter estimates are computed as a mean of a number of realizations from this chain. Details on the implementation of this stochastic EM algorithm are in Appendix E.

4.4 Model fit

In Appendix Figure F1, I plot the complete-data likelihood of over iterations of the second-stage EM-based estimation. We see that despite only having a few iterations, the complete-data likelihood seems to have converged. In this section, I discuss the estimates of the structural parameters: demand elasticities and production function parameters. Then, I assess the fit of the model in its ability to match cross-sectional and dynamic features of output and wages.

without demand or labor market advantages is discussed in Aguirre et al. (2021) and Doty (2022).

In Appendix Figure F2, I report the estimated demand elasticity and corresponding bootstrap distribution from the first-stage GMM estimation. I find point estimates that are negative and slightly greater than one in absolute value. This would suggest that the firms operate in the elastic portion of their demand curves. In Appendix Table F1, I report the estimated production function parameters. Moreover, I report the sum of the input elasticities and report it as a measure of returns to scale. I find returns to scale measure close to one suggesting that firms operate with constant returns to scale (at least locally) which is plausible given my sample tends to lean towards larger firms. Moreover, the data is only available at the firm-level and not at the establishment-level.

In Panels (a)–(c) of Appendix Figure F3, I plot the marginal distribution of output in the data and based on the model. The differences in the marginal distributions are imperceptible. Though we fit the cross-sectional distribution of output well, we might be concerned that we do not fit the dynamics well. Thus, in Panels (d)–(f) of the same figure, I plot the persistence in output in the data, based on a flexible quantile specification of the dynamics, which we can compare to Panels (g)-(i) which are corresponding persistence measures in output based on the model. We find that the model is able to match the persistence patterns in output well.

In Panels (a)–(c) of Appendix Figure F4, I instead plot the the marginal distribution of wages from the data and the model. Comparing the two, we again find that we fit the cross-sectional distribution of wages well. In Panels (d)–(f) and (g)–(i) of the same figure, we plot the persistence of wages based on the data and the model, respectively. The model has a lower level of persistence in wages, in general. However, we match well the general persistence patterns over initial wages and the size of the shock.

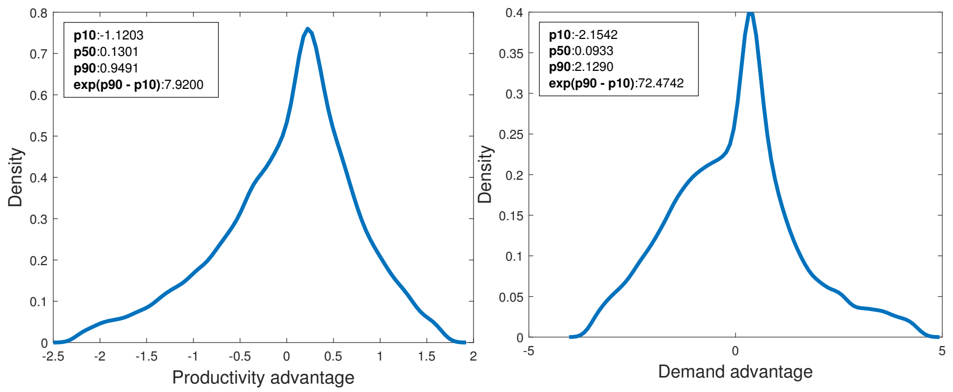
5 Cross-sectional distribution and dynamics of productivity, demand, and labor market advantages

In this section, I document empirical facts on the cross-sectional distribution, dynamics, and joint distribution of productivity, demand advantages, and labor market advantages.

5.1 Cross-sectional heterogeneity

Figure 1: Cross-sectional distribution of productivity and demand advantages

(a) Productivity (b) Demand advantages



Notes: Panel (a) and (b) present estimates of the cross-sectional distributions of productivity and demand advantages respectively. Also reported are the 10th, 50th, and 90th percentiles of the distributions. Dispersion of the distributions measured as $\exp(P90 - P10)$ *also reported. Kernel densities estimated on data with top and bottom 2% trimmed. Plots for individual sectors are presented in Appendix Figure G1. Percentiles computed with full data. Distributions of individual sectors weighted by sector sales.*

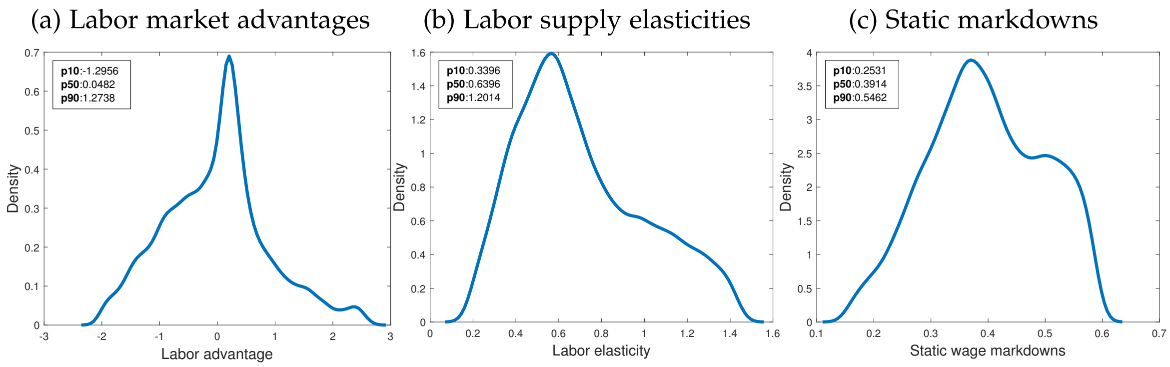
Productivity and demand advantages. Productivity differences among firms have been extensively studied in the literature. Figure 1 shows estimates of the marginal distributions of productivity and demand advantages. Both show substantial deviations from Gaussianity with negative skewness and fat tails. Focusing on productivity in Panel (a), we find significant dispersion in productivity across firms—given the same inputs, the firm at the 90th percentile produces about 8 times more than the firm at the 10th percentile of the productivity distribution. Comparing this to estimates in Hsieh and Klenow (2009), I find that the cross-sectional dispersion of productivity in Portugal is smaller than that in the US, China, and India.[22]

On the other hand, focusing on Panel (b) which plots the density of demand advantages, we similarly find economically significant heterogeneity across firms—fixing prices, quantity demanded of the firm at the 90th percentile of the demand advantage distribution is about 72 times that of the firm at the 10th percentile of

[22]Specifically, I compare it to their estimates of the cross-sectional dispersion in $\ln(TFPQ)$.

the distribution. This magnitude may seem extremely large but firms at different parts of the demand advantage distribution would optimally choose different prices which will affect the actual distribution of quantity demanded.

Figure 2: Labor market advantages, labor supply elasticity, and implied static markdowns

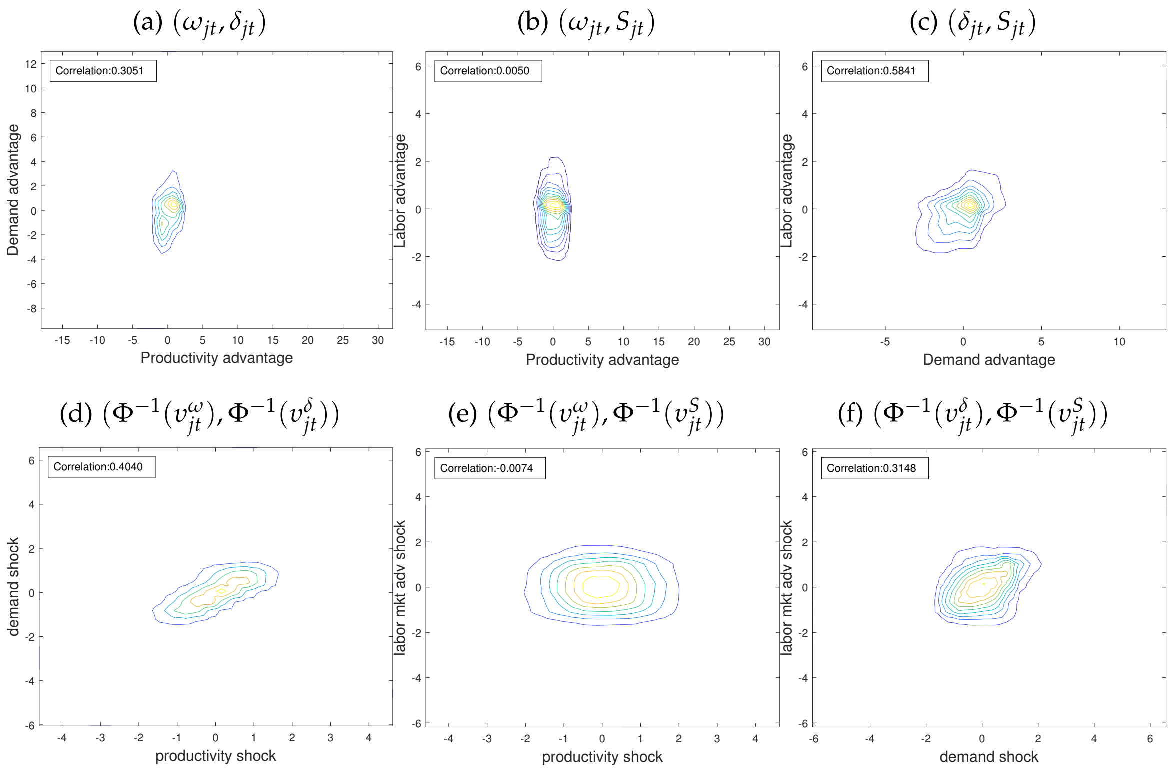


Notes: Estimates of the marginal distributions of labor market advantages (Panel (a)), labor supply elasticities (Panel (b)), and static markdowns (Panel (c)). Labor supply elasticites measured as the derivative of the labor supply function in Equation (14) with respect to $\ln w_{jt}$. *Static wage markdown measured as* $\varepsilon_{jt}^{\ell w}/(1+\varepsilon_{jt}^{\ell w})$ *where* $\varepsilon_{jt}^{\ell w}$ *is the labor supply elasticities of wages. They are interpreted as proportions of marginal revenue product of labor paid to workers in the form of wages. Kernel densities estimated on data with top and bottom 2% trimmed. Percentiles computed with full data. Plots for individual sectors are presented in Appendix Figure G2. Distributions of individual sectors weighted by sector sales.*

Labor market advantages. In Panel (a) of Figure 2, we plot the estimated density of labor market advantages. However, as labor market advantages S_{jt} enter the residual labor supply function faced by the firm in a nonlinear manner, as in Equation (14), it requires a more nuanced interpretation. In the model, labor market advantages may affect labor supply elasticities faced by firms. In Panel (b) of the same figure, we find the marginal density of labor supply elasticities across firms. We find that the distribution may be multi-modal owing to different industries being aggregated in the figure. Overall, we find that elasticities range between 0.34 at the 10th percentile and 1.20 at the 90th percentile.

As firms face differing labor supply elasticities, they also differ in their labor market power. In the absence of wage adjustment costs, firms pay wages that are markdowns of marginal revenue product of labor (MRPL). The exact markdown is determined by the labor supply elasticity. In Panel (c) of Figure 2, we see the density of static markdowns. The median of the distribution is abour 0.39 suggesting that the firm pays about 39% of MRPL. However, markdowns range between 0.25 to 0.54. In the presence of wage adjustment costs, as is the case in the framework, markdowns are likely to be larger (paying more of MRPL) due to the costs to adjust wages and its dynamic implications (Seegmiller, 2021).

Figure 3: Joint density of firm heterogeneity and shocks



Notes: Panels (a)–(c) are contour plots of the estimated joint distributions of productivity and demand advantages, productivity and labor market advantages, and demand and labor market advantages, respectively. Panels (d)–(f) are contours of the estimated copula densities of the shocks to productivity (v^{ω}_{jt}), demand (v^{δ}_{jt}), and labor market advantages (v^{S}_{jt}). As a graphical convention, I rescale the marginals of the shocks so they are standard normal. Correlations also reported. Plots for individual sectors are presented in Appendix Figures G3–G5. Distributions of individual sectors weighted by sector sales.

Joint distribution. In the model, we allow for correlation in the different dimensions of firm heterogeneity. In Panels (a)–(c) of Figure 3, we plot the bivariate joint densities of productivity, demand, and labor market advantages. We find no correlation between productivity and labor market advantages. There is positive correlation between productivity and demand advantages, and an even stronger positive correlation between demand and labor market advantages.

These correlations are partially explained by the correlation in the shocks as defined in Equations (4)–(6). We estimate the univariate quantile processes from simulations of the triangular model to back out shocks of the univariate processes. In Panels (d)–(f) of Figure 3, we plot contour plots of the copula between these shocks. The shocks to productivity and labor market advantages are not correlated. However, we find strong correlation between productivity and demand shocks, as well as demand and labor market advantage shocks.

5.2 Dynamics: Persistence and conditional distribution of shocks

This framework allows for rich dynamics for productivity, demand advantages, and labor market advantages. Consider the univariate quantile models we described in Equations (4)–(6). For $\tau \in (0,1)$ and $z \in \{\omega, \delta, S\}$,

$$\rho(z_{jt-1}, \tau) = \frac{\partial Q^z(z_{jt-1}, \tau)}{\partial z} \tag{29}$$

is a measure of the persistence of histories—it measures the persistence of z_{it-1} when it is hit by a shock of rank τ. Low persistence for a particular type of shock would suggest that shock is able to wipe out the memory of past shocks. In canonical models where productivity is modeled as a linear $AR(1)$ process, then $\rho(z_{it-1}, \tau)$ is constant—that is, it does not depend on z_{it-1} nor τ— and is equal to the autoregressive parameter.

As the model does not restrict the conditional distribution of z_{it} given z_{it-1}, it allows for arbitrary conditional heteroskedasticity and skewness. That is, the dispersion and asymmetry of the distribution of shocks may depend on the previous level of productivity, demand, or labor market advantages. The following quantities

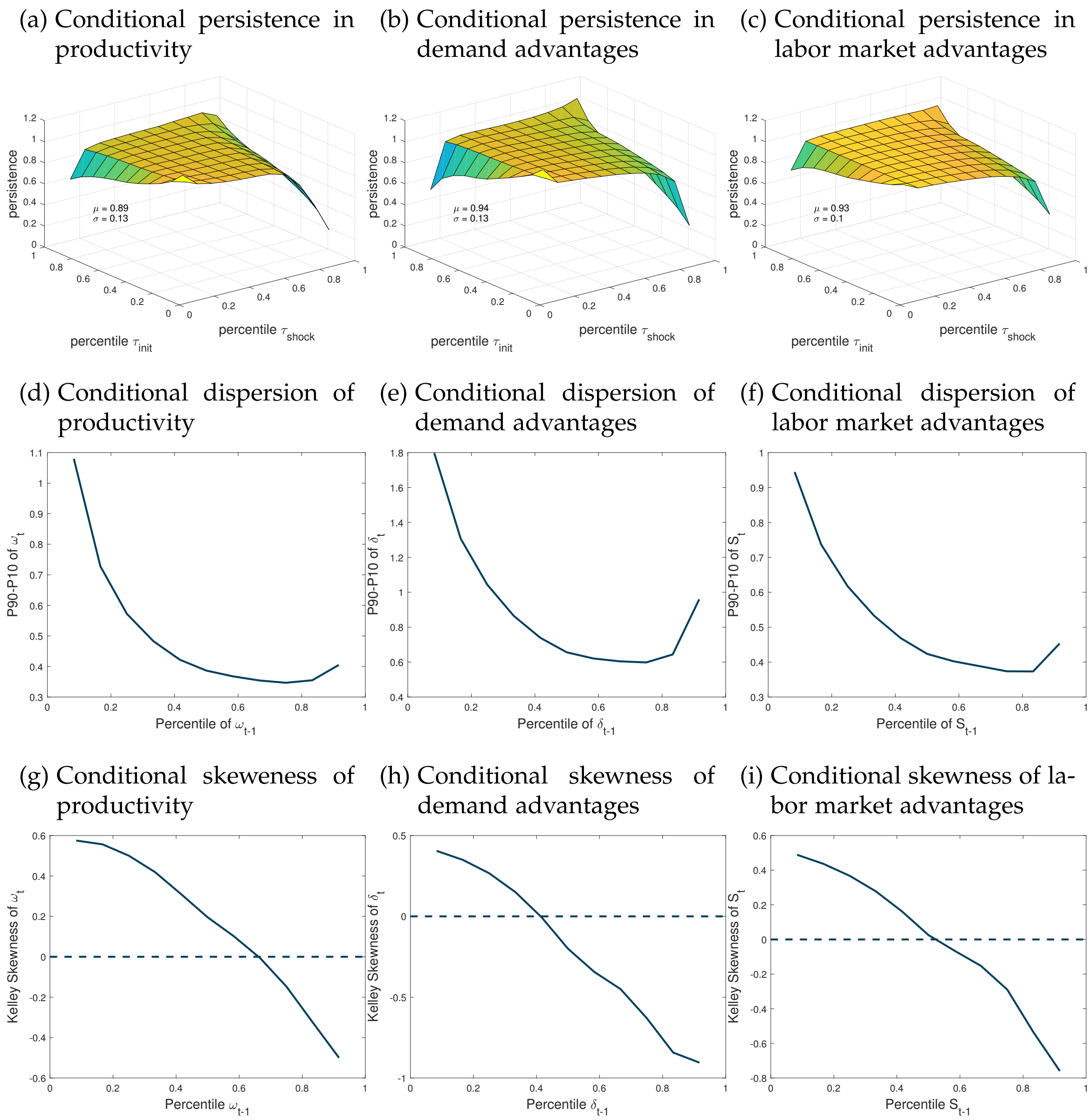
$$\sigma^z(z_{jt-1}) = Q^z(z_{jt-1}, 0.9) - Q^z(z_{jt-1}, 0.1) \tag{30}$$

$$skew^z(z_{jt-1}) = \frac{[Q^z(z_{jt-1}, 0.9) - Q^z(z_{jt-1}, 0.5)] - [Q^z(z_{jt-1}, 0.5) - Q^z(z_{jt-1}, 0.1)]}{Q^z(z_{jt-1}, 0.9) - Q^z(z_{jt-1}, 0.1)} \tag{31}$$

are quantile-based measures of the conditional dispersion and conditional skewness of z_{it} given z_{it-1}, respectively. As we condition on the past level z_{it-1}, we can interpret features of this conditional distribution as the distribution of a "shock". Intuitively, the conditional dispersion of z_{it} given z_{it-1} captures how much uncertainty there is the stochastic process. Allowing this to differ based on z_{it-1} would suggest that firms in different parts of the distribution may face different levels of uncertainty about the future. On the other hand, conditional skewness is a measure of asymmetry. The measure compares how much of overall dispersion ($P90 - P10$) is in the right tail ($P90 - P50$) compared to the left tail ($P50 - P10$). If the skewness measure is negative, it would suggest that bad shocks could be worse in magnitude than good shocks. Thus, non-constant conditional skewness would suggest the asymmetry of shocks depend on where a firm is on the overall distribution. In the canonical $AR(1)$ model usually employed for firm dynamics (i.e., with Gaussian innovations), the distribution of shocks are Normally distributed with constant variance. Conditional dispersion would be constant and conditional skewness is zero—that is, uncertainty is constant across all firms and that there are no asymmetry in the magnitude of shocks.

Nonlinear persistence. In Panels (a)–(c) of Figure 4, I plot the persistence of productivity, demand, and labor market advantages, respectively, as measured in Equation (29). These graphs show the persistence as they vary over the level of productivity, demand, or labor market advantage (measured as percentiles relative to sector distributions, τ_{init}) and the size of the shock (also measured as percentiles, τ_{shock}). First, we notice that the surfaces are not flat and constant which would suggest that persistence depends not only on the level of productivity, demand, and labor market advantages but also on the size and direction of the shock. This is a deviation from the canonical $AR(1)$ model where persistence is constant regardless of the past and of the size and direction of the shock. Second, we notice that the persistence of productivity is generally lower than that of demand or labor market advantages. Lastly, the shape of the surface would suggest that large positive shocks tend to erase the history of bad firms (measured by their position in the distribution). This would suggest, for instance, that there are shocks that push historically low productivity firms to higher productivity states in the future. We can imagine, as an example, a firm hiring a new manager who revamps the production process to be more efficient.

Figure 4: Dynamics of productivity, demand, and labor market advantages



(a) Conditional persistence in productivity

(b) Conditional persistence in demand advantages

(c) Conditional persistence in labor market advantages

(d) Conditional dispersion of productivity

(e) Conditional dispersion of demand advantages

(f) Conditional dispersion of labor market advantages

(g) Conditional skeweness of productivity

(h) Conditional skewness of demand advantages

(i) Conditional skewness of labor market advantages

Notes: Panels (a)–(c) are estimates of the persistence of productivity, demand, and labor market advantages, respectively, conditional on the percentile of the past state (τ_{init}) and percentile of the shock (τ_{shock}). They are obtained as estimates of the average derivative of the conditional quantile function of the state z_{jt} given the previous state z_{jt-1} with respect to z_{jt-1}. Panels (d)–(f) present estimates of the conditional dispersion of the states given past states measured as the $P90 - P10$ of the predictive distribution. Panel (g)–(j) present estimates of the conditional skewness of the states given past states measured as $\frac{(P90-P50)-(P50-P10)}{P90-p10}$ of the predictive distribution. Plots for individual sectors are presented in Appendix Figures G6–G8. Distributions of individual sectors weighted by sector sales.

Previous work using alternative datasets have found the same patterns for productivity. We find the same patterns in Portuguese data. However, we show that demand and labor market advantages follow similar dynamics.

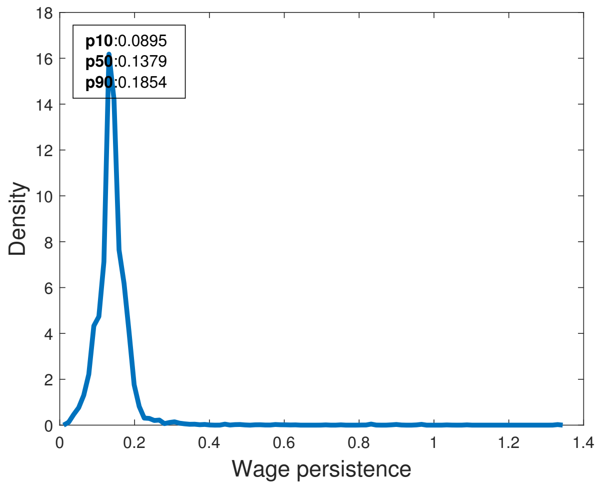
Conditional distributions. Our models allow for the conditional distribution of future productivity, demand, or labor market advantages given their corresponding levels unrestricted. Looking at this conditional distribution gives us an idea of the distribution of shocks, and consequently, of uncertainty. In Panels (d)–(f) of Figure 4, I plot the conditional dispersion of firm heterogeneity conditional on its past. In general, we find that the conditional dispersion of firm heterogeneity decreases as you go up the distribution. This suggests that, for instance, firms at the top of the productivity distribution have less uncertainty about future productivity than firms at the bottom of the distribution.

Dispersion does not tell the full story—firms also care about whether the uncertainty might be in their favor or not. In Panels (g)–(i) of Figure 4, I plot the conditional skewness of the distribution that speaks towards the asymmetry of shocks. We see a common picture: at the lower end of the distribution, there is positive skewness whereas there is negative skewness at the top of the distribution. This gives us a more full picture of the uncertainty faced by firms. Speaking more loosely, focusing again on productivity, the conditional dispersion result tells us that firms at the lower end of the productivity tend to have higher uncertainty, in general; however, the conditional skewness results tell us that this increased uncertainty is skewed towards positive deviations. Firms at the lower end of the productivity distribution have can expect future outcomes to be from a wider range of values but what the range of good shocks are large than the range of bad ones. On the other end of the distribution, firms with high productivity have less uncertainty about future outcomes but bad shocks tend to be worse than what they stand to benefit from good shocks.

Combining these results with the results on conditional persistence, it would seem that firms at the bottom of the distribution has more room to improve than to fall while the firms at the top of the distribution have more room to fall and less room to move up. This result may seems obvious but it is important to reiterate that the standard $AR(1)$ model used to describe dynamics do not allow for this and instead paint a very different picture of uncertainty where all firms have the equal change of moving up or falling.

6 Wage dynamics: adjustment costs and pass-through of shocks

Figure 5: Autocorrelation in wages



Notes: Marginal distribution of the derivative of the wage-setting equation given past log-wages $\ln w_{jt-1}$ *and other state variables (capital* k_{jt}*, productivity* ω_{jt}*, demand* δ_{jt}*, and labor market advantages* S_{jt}*) with respect to* $\ln w_{jt-1}$*. Kernel densities estimated on data with top and bottom 2% trimmed. Percentiles computed with full data. Plots for individual sectors are presented in Appendix Figure G9. Distributions of individual sectors weighted by sector sales.*

Wage persistence and adjustment costs. In the absence of adjustment costs, wages should be exactly determined by capital, productivity, demand advantages, and labor market advantages. Past realizations of wages should be independent of current wages conditional on the state variables. Thus, any remaining autocorrelation in wages after controlling for the state variables would suggest the presence of wage adjustment costs. In Figure 5, I plot the empirical density of the derivative of the wage equation in Equation (7) with respect to lagged wages. We find that the remaining autocorrelation in wages after controlling for the state variables of the firm varies between 0.09 to 0.19 and has a long right tail. At the median, it is about 0.14 which is slightly smaller compared to the estimates obtained by Carneiro et al. (2022) in a worker-level regression of wages on its lag, controlling for time-invariant worker, employer, and match fixed effects using the same matched employer-employee dataset used in this paper.[23] The role played by lagged wages in the wage-setting decision of the firm would suggest the presence of economically-relevant wage adjustment costs that have implications on the pass-through of firm shocks to wages.

Table 1: Wage pass-through of productivity and demand shocks

		Labor Market Advantage		
	Overall (1)	p10 (2)	p50 (3)	p90 (4)
Productivity shock p90	−0.0023	−0.0072	−0.0021	0.0000
Productivity shock p10	−0.0027	−0.0076	−0.0021	0.0001
Demand shock p90	0.0149	−0.0002	0.0191	0.0117
Demand shock p10	0.0210	0.0230	0.0233	0.0233

Notes: Median estimates of wage pass-through elasticities. P90 and P10 are interpreted as good and bad shocks, respectively. Columns (2)–(4) report the conditional medians given position in the labor market advantage distribution. Based on distributions of individual sectors weighted by sector sales.

Wage pass-through of productivity and demand shocks. We are interested in measuring the pass-through of productivity and demand shocks onto wages and how labor market advantages mediate this. With the framework we have built, we are able to measure the pass-through of positive and negative shocks separately to see whether there is symmetry in the pass-through. To measure wage pass-through, suppose of a positive productivity shock, we consider the following thought exercise: for each firm, we consider a scenario where that firm receives a positive shock (quantified as a shock at the 90th percentile) and compare it to the scenario where the same firm receives a neutral shock (a shock at the median). We compare the change in wages and scale it by the change in productivity to obtain

[23] In their most demanding specification that includes match effects, Carneiro et al. (2022) report an autocorrelation of arount 0.22 (and around 0.29 after bias-correction). Using survey data in the US (PSID), Hospido (2015) estimates the autocorrelation in wages after controlling for individual and job fixed heterogeneity and finds a coefficient of around 0.07. A notable difference between my exercise and theirs is that they focus on worker-level autocorrelation in wages while I focus on the autocorrelation in average firm-level wages controlling for composition. As such, it is more comparable to the exercise by Hospido (2015) where they condition on job stayers and find an autocorrelation of about 0.09.

a measure that is an elasticity. Mathematically, it is the following quantity:

$$PT_{jt}(0.9) = \frac{\ln Q^{w}(w_{jt-1}, K_{jt}, Q^{\omega}(\omega_{t-1}, 0.9), \delta_{jt}, S_{jt}) - \ln Q^{w}(w_{jt-1}, K_{jt}, Q^{\omega}(\omega_{t-1}, 0.5), \delta_{jt}, S_{jt})}{Q^{\omega}(\omega_{t-1}, 0.9) - Q^{\omega}(\omega_{t-1}, 0.5)} \tag{32}$$

Similar quantities can be defined with a negative productivity shock, and for demand shocks. The marginal distribution of this object is interesting as it tell us about wage pass-through in general. However, we can also look at this pass-through conditioning on firm characteristics.

In Table 1, I report median wage pass-through estimates for good and bad productivity and demand shocks. Focusing on Column (1), I highlight three things. First, the pass-through estimates for demand shocks are larger than the pass-through of productivity shocks. In fact, the magnitude of pass-through for productivity shocks is close to zero and is economically insignificant. The difference in the persistence of productivity from that of demand partially accounts for this difference in pass-through. As mentioned, demand shocks are more persistent so we would expect them to be passed onto workers more. The second thing I want to highlight is that there is suggestion of asymmetry between the pass-through of good and bad demand shocks. Based on the point estimates, the pass-through of bad demand shocks is larger than the pass-through of good demand shocks. However, we do not have confidence intervals to assess the statistical significance of this difference.[24] Lastly, I want to comment on the magnitude of the pass-through elasticities. The estimates of the pass-through elasticities for demand shocks are smaller than the estimates obtained for the pass-through of revenue or value-added shocks, for instance, found by Guiso et al. (2005) and Cardoso and Portela (2009).

In Columns (2)–(4) of Table 1, I examine the heterogeneity of the wage pass-through elasticity with respect to labor market advantages, looking at firms at the 10th percentile of the labor market advantage distribution, at the median, and at the 90th percentile. Consistent with the results without heterogeneity, I do not find quantitatively significant wage pass-through of productivity shocks. The wage pass-through elasticities of bad demand shocks appear to be constant across the distribution of labor market advantages. On the other hand, I find that the pass-through of good demand shocks depend on the level of labor market advantages:

[24] Under correct specification, nonparametric and parametric block-boostrap confidence intervals will have asymptotic validity with fixed T. This is a computationally-intensive exercise and are not yet reported in the current version of this paper.

firms which have larger labor market advantages tend to have larger wage pass-through elasticities of good demand shocks.

7 Conclusion

In this paper, I build and estimate an empirical framework of firms and workers in imperfectly competitive output and labor markets to measure the pass-through of productivity and demand shocks to wages. This leverages a unique data from Portugal that combines matched employer-employee data, financial statements data, and a manufacturing survey. The productivity, demand, and labor market advantages of firms are inferred from the observed data. Matched employee-employer data provides us information to control for worker heterogeneity in within-firm wages. On the other hand, the separate price and quantity data allows us to separate productivity from demand shocks.

I find substantial cross-sectional heterogeneity in productivity, demand, and labor market advantages. Productivity and demand advantages are positively correlated and so are demand and labor market advantages. These joint correlations are partially explained by the correlation in shocks as well as the correlation in initial conditions. The three features of the firm evolve in rich, nonlinear ways. In particular, there are positive shocks to poor-performing firms that reduce the persistence of its past states. The predictive distribution of states also depend on the current state. Firms at the bottom of the distribution of states face more uncertainty that tends to be positively skewed. These features of the dynamics of productivity, demand, and labor market advantages have implications on many firm decisions.

I measure the pass-through of productivity and demand shocks to wages. I find that the pass-through of productivity and demand shocks are asymmetric—I find a positive pass-through of demand shocks but no pass-through of productivity shocks, after accounting for wage adjustment costs. Moreover, I find suggestive evidence that the pass-through of bad demand shocks are larger than the pass-through of good demand shocks. The wage pass-through elasticities of demand shocks I estimate are smaller compared to the estimates of the pass-through of revenue or value-added shocks. The wage pass-through of good demand shocks depend on the firm's position in the labor market. Firms which have higher labor market advantages—i.e., firms more attractive to workers conditional on wages—

have higher pass-through elasticities of good demand shocks than firms are the bottom of the labor market advantage distribution. I do not find this difference in the pass-through of bad demand shocks. This suggests interactions between firms' output and labor market power.

The framework in this paper can be extended and applied to study other topics of interest in firm dynamics and labor economics. For instance, we might be interested in whether the pass-through of wages differ across workers within the same firm. As previously discussed, the framework can be easily extended if we are interested in differences of pass-through across observable heterogeneity across workers (e.g., by educational attainment of workers or gender). Another exciting avenue for further research is endogenizing productivity, demand, and labor market advantages to study how firm-level shocks affect decisions of investing in innovation research, advertising, and amenities.

References

Abowd, John A. and Thomas Lemieux (1993) "The Effects of Product Market Competition on Collective Bargaining Agreements: The Case of Foreign Competition in Canada," *The Quarterly Journal of Economics*, 108 (4), 983–1014, http://www.jstor.org/stable/2118457. (cited in page 6)

Abowd, John M., Francis Kramarz, and David N. Margolis (1999) "High Wage Workers and High Wage Firms," *Econometrica*, 67 (2), 251–333, https://doi.org/10.1111/1468-0262.00020. (cited in page 1)

Ackerberg, Daniel (2020) "Timing Assumptions and Efficiency: Empirical Evidence in a Production Function Context," Working Paper. (cited in page 8)

Ackerberg, Daniel A., Kevin Caves, and Garth Frazer (2015) "Identification Properties of Recent Production Function Estimators," *Econometrica*, 83 (6), 2411–2451, 10.3982/ECTA13408. (cited in page 6, 8, 19)

Adamopoulou, Effrosyni and Ernesto Villanueva (2022) "Wage determination and the bite of collective contracts in Italy and Spain," *Labour Economics*, 76, 102147, https://doi.org/10.1016/j.labeco.2022.102147. (cited in page 12)

Aguirre, Alvaro, Matias Tapia, and Lucciano Villacorta (2021) "Production, Investment and Wealth Dynamics Under Financial Frictions: An Empirical Investigation of the Self-Financing Channel," Working Paper. (cited in page 9, 25)

Amemiya, Takeshi (1977) "The Maximum Likelihood and the Nonlinear Three-Stage Least Squares Estimator in the General Nonlinear Simultaneous Equation Model," *Econometrica*, 45 (4), 955–968, http://www.jstor.org/stable/1912684. (cited in page A14)

Arellano, Manuel, Richard Blundell, and Stéphane Bonhomme (2017) "Earnings and Consumption Dynamics: A Nonlinear Panel Data Framework," *Econometrica*, 85 (3), 693–734, 10.3982/ECTA13795. (cited in page 3, 8, 20, 21, 22, 24, 25, A16, A17)

Arellano, Manuel and Stephen Bond (1991) "Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations," *The Review of Economic Studies*, 58 (2), 277–297, 10.2307/2297968. (cited in page 8)

Arellano, Manuel and Stéphane Bonhomme (2016) "Nonlinear panel data estimation via quantile regressions," *The Econometrics Journal*, 19 (3), C61–C94, 10.1111/ectj.12062. (cited in page 8, 22, 25, A15, A16)

——— (2017) "Nonlinear Panel Data Methods for Dynamic Heterogeneous Agent Models," *Annual Review of Economics*, 9 (1), 471–496, 10.1146/annurev-economics-063016-104346. (cited in page 8, 24)

Blundell, Richard and Stephen Bond (1998) "Initial conditions and moment restrictions in dynamic panel data models," *Journal of Econometrics*, 87 (1), 115 – 143, https://doi.org/10.1016/S0304-4076(98)00009-8. (cited in page 8)

——— (2000) "GMM Estimation with persistent panel data: an application to production functions," *Econometric Reviews*, 19 (3), 321–340, 10.1080/07474930008800475. (cited in page 8)

Blundell, Richard, Luigi Pistaferri, and Ian Preston (2008) "Consumption Inequality and Partial Insurance," *American Economic Review*, 98 (5), 1887–1921, 10.1257/aer.98.5.1887. (cited in page 9)

Bond, Steve, Arshia Hashemi, Greg Kaplan, and Piotr Zoch (2021) "Some Unpleasant Markup Arithmetic: Production Function Elasticities and their Estimation from Production Data," *Journal of Monetary Economics*, https://doi.org/10.1016/j.jmoneco.2021.05.004. (cited in page 7)

Bover, Olympia, Pilar García-Perea, and Pedro Portugal (2000) "Labour market outliers: Lessons from Portugal and Spain," *Economic Policy*, 15 (31), 380–428, 10.1111/1468-0327.00065. (cited in page 12)

Bronnenberg, Bart J., Sanjay K. Dhar, and Jean-Pierre H. Dubé (2007) "Consumer Packaged Goods in the United States: National Brands, Local Branding," *Journal of Marketing Research*, 44 (1), 4–13, 10.1509/jmkr.44.1.004. (cited in page 15)

——— (2009) "Brand History, Geography, and the Persistence of Brand Shares," *Journal of Political Economy*, 117 (1), 87–115, 10.1086/597301. (cited in page 15)

de Burton, Simon (2021) "The enduring appeal of the Vespa," *The Spectator*, `https://www.spectator.co.uk/article/75-years-on-the-vespa-is-still-going-strong`. (cited in page 15)

Caliendo, Lorenzo, Giordano Mion, Luca David Opromolla, and Esteban Rossi-Hansberg (2020) "Productivity and Organization in Portuguese Firms," *Journal of Political Economy*, 128 (11), 4211–4257, 10.1086/710533. (cited in page 10)

Card, David and Ana Rute Cardoso (2021) "Wage flexibility under sectoral bargaining," Working Paper. (cited in page 12)

Card, David, Ana Rute Cardoso, Joerg Heining, and Patrick Kline (2018) "Firms and Labor Market Inequality: Evidence and Some Theory," *Journal of Labor Economics*, 36 (S1), S13–S70, 10.1086/694153. (cited in page 3, 16)

Cardoso, Ana Rute and Miguel Portela (2009) "Micro Foundations for Wage Flexibility: Wage Insurance at the Firm Level," *The Scandinavian Journal of Economics*, 111 (1), 29–50, 10.1111/j.1467-9442.2008.01553.x. (cited in page 4, 5, 10, 36)

Cardoso, Ana Rute and Pedro Portugal (2005) "Contractual Wages and the Wage Cushion under Different Bargaining Settings," *Journal of Labor Economics*, 23 (4), 875–902, 10.1086/491608. (cited in page 12)

Carlsson, Mikael, Julián Messina, and Oskar Nordström Skans (2020) "Firm-Level Shocks and Labour Flows," *The Economic Journal*, 131 (634), 598–623, 10.1093/ej/ueaa087. (cited in page 7)

Carlsson, Mikael, Julián Messina, and Oskar Nordström Skans (2016) "Wage Adjustment and Productivity Shocks," *The Economic Journal*, 126 (595), 1739–1773, 10.1111/ecoj.12214. (cited in page 7)

Carneiro, Anabela, Pedro Portugal, Pedro Raposo, and Paulo M.M. Rodrigues (2022) "The persistence of wages," *Journal of Econometrics*, https://doi.org/10.1016/j.jeconom.2021.11.014. (cited in page 34, 35)

Chan, Mons, Sergio Salgado, and Ming Xu (2019) "Heterogeneous Passthrough from TFP to Wages," Working Paper. (cited in page 1, 6, 14)

Cho, David (2018) "The Labor Market Effects of Demand Shocks: Firm-Level Evidence from the Recovery Act," Working Paper. (cited in page 6)

De Loecker, Jan and Chad Syverson (2021) "An Industrial Organization Perspective on Productivity," Working Paper. (cited in page 6)

De Loecker, Jan and Frederic Warzynski (2012) "Markups and Firm-Level Export Status," *American Economic Review*, 102 (6), 2437–71, 10.1257/aer.102.6.2437. (cited in page 7)

Demirer, Mert (2019) "Production Function Estimation with Factor-Augmenting Technology: An Application to Markups," Working Paper. (cited in page 14)

Dempster, A. P., N. M. Laird, and D. B. Rubin (1977) "Maximum Likelihood from Incomplete Data via the EM Algorithm," *Journal of the Royal Statistical Society. Series B (Methodological)*, 39 (1), 1–38, `http://www.jstor.org/stable/2984875`. (cited in page A16)

Doraszelski, Ulrich and Jordi Jaumandreu (2018) "Measuring the Bias of Technological Change," *Journal of Political Economy*, 126 (3), 1027–1084, 10.1086/697204. (cited in page 14)

Doty, Justin (2022) "A Dynamic Framework for Identification and Estimation of Nonseparable Production Functions," Working Paper. (cited in page 25)

Eslava, Marcela, John Haltiwanger, Adriana Kugler, and Maurice Kugler (2004) "The effects of structural reforms on productivity and profitability enhancing reallocation: evidence from Colombia," *Journal of Development Economics*, 75 (2), 333 – 371, https://doi.org/10.1016/j.jdeveco.2004.06.002, 15th Inter American Seminar on Economics. (cited in page 7)

Foster, Lucia, John Haltiwanger, and Chad Syverson (2008) "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?" *American Economic Review*, 98 (1), 394–425, 10.1257/aer.98.1.394. (cited in page 7, 13)

Friedrich, Benjamin, Lisa Laun, Costas Meghir, and Luigi Pistaferri (2019) "Earnings Dynamics and Firm-Level Shocks," Working Paper. (cited in page 6)

Gálvez, Julio (2017) "Household portfolio choices and nonlinear income risk," Working Paper. (cited in page 9)

Gandhi, Amit, Salvador Navarro, and David A. Rivers (2020) "On the Identification of Gross Output Production Functions," *Journal of Political Economy*, 128 (8), 2973–3016, 10.1086/707736. (cited in page 8, A8)

Garin, Andrew and Felipe Silverio (2018) "How Responsive are Wages to Demand within the Firm? Evidence from Idiosyncratic Export Demand Shocks," Working Paper. (cited in page 1, 6)

Goldberg, Pinelopi Koujianou (1995) "Product Differentiation and Oligopoly in International Markets: The Case of the U.S. Automobile Industry," *Econometrica*, 63 (4), 891–951, http://www.jstor.org/stable/2171803. (cited in page 15)

Guadalupe, Maria (2007) "Product Market Competition, Returns to Skill, and Wage Inequality," *Journal of Labor Economics*, 25 (3), 439–474, 10.1086/513299. (cited in page 6)

Guertzgen, N. (2014) "Wage insurance within German firms: do institutions matter?" *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, 177 (2), 345–369, 10.1111/rssa.12019. (cited in page 5)

Guillard, Charlotte, Jordi Jaumandreu, and Jocelyn Olivari (2018) "Endogenous Productivity and Unobserved Prices," Working Paper. (cited in page 7)

Guiso, Luigi, Luigi Pistaferri, and Fabiano Schivardi (2005) "Insurance within the Firm," *Journal of Political Economy*, 113 (5), 1054–1087, 10.1086/432136. (cited in page 1, 4, 5, 6, 10, 36)

Heckman, James J. and Guilherme Sedlacek (1985) "Heterogeneity, Aggregation, and Market Wage Functions: An Empirical Model of Self-Selection in the Labor Market," *Journal of Political Economy*, 93 (6), 1077–1125, 10.1086/261352. (cited in page 18)

Hospido, Laura (2015) "Wage dynamics in the presence of unobserved individual and job heterogeneity," *Labour Economics*, 33, 81–93, https://doi.org/10.1016/j.labeco.2015.03.012. (cited in page 35)

Hotelling, Harold (1929) "Stability in Competition," *The Economic Journal*, 39 (153), 41–57, http://www.jstor.org/stable/2224214. (cited in page 15)

Hsieh, Chang-Tai and Peter J. Klenow (2009) "Misallocation and Manufacturing TFP in China and India," *The Quarterly Journal of Economics*, 124 (4), 1403–1448, 10.1162/qjec.2009.124.4.1403. (cited in page 27)

Hu, Yingyao and Susanne M. Schennach (2008) "Instrumental Variable Treatment of Nonclassical Measurement Error Models," *Econometrica*, 76 (1), 195–216, https://doi.org/10.1111/j.0012-9682.2008.00823.x. (cited in page 8, 24)

Hu, Yingyao and Matthew Shum (2012) "Nonparametric identification of dynamic models with unobserved state variables," *Journal of Econometrics*, 171 (1), 32–44, https://doi.org/10.1016/j.jeconom.2012.05.023. (cited in page 24)

Jaumandreu, Jordi and Heng Yin (2019) "Cost and Product Advantages: Evidence from Chinese Manufacturing Firms," Working Paper. (cited in page 7, 8, 20)

Juhn, Chinhui, Kristin McCue, Holly Monti, and Brooks Pierce (2018) "Firm Performance and the Volatility of Worker Earnings," *Journal of Labor Economics*, 36 (S1), S99–S131, 10.1086/694167. (cited in page 5)

Kátay, Gábor (2016) "Do Firms Provide Wage Insurance against Shocks?" *The Scandinavian Journal of Economics*, 118 (1), 105–128, 10.1111/sjoe.12128. (cited in page 1, 6)

Klette, Tor Jakob and Zvi Griliches (1996) "The inconsistency of common scale estimators when output prices are unobserved and endogenous," *Journal of Applied Econometrics*, 11 (4), 343–361, 10.1002/(SICI)1099-1255(199607)11:4<343::AID-JAE404>3.0.CO;2-4. (cited in page 6)

Kumar, Pradeep and Hongsong Zhang (2019) "Productivity or Demand Shocks: What Determines Firms' Investment and Exit Decisions?" *International Economic Review*, 60 (1), 303–327, 10.1111/iere.12354. (cited in page 7)

Lamadon, Thibaut (2016) "Productivity Shocks, Long-Term Contracts and Earnings Dynamics," Working Paper. (cited in page 1, 6)

Lamadon, Thibaut, Magne Mogstad, and Bradley Setzler (2019) "Imperfect Competition, Compensating Differentials and Rent Sharing in the US Labor Market," Working Paper. (cited in page 6, 16)

Levinsohn, James and Amil Petrin (2003) "Estimating Production Functions Using Inputs to Control for Unobservables," *The Review of Economic Studies*, 70 (2), 317–341, 10.1111/1467-937X.00246. (cited in page 6, 8, 19)

Lucas, Robert E, Jr. (1976) "Econometric policy evaluation: A critique," in *Carnegie-Rochester conference series on public policy*, 1, 19–46, North-Holland. (cited in page 13)

Marques, Carlos, Fernando Martins, and Pedro Portugal (2010) "Price and wage formation in Portugal,"Technical report, European Central Bank. (cited in page 18)

Miller, Nathan H. and Matthew Osborne (2014) "Spatial differentiation and price discrimination in the cement industry: evidence from a structural model," *The RAND Journal of Economics*, 45 (2), 221–247, http://www.jstor.org/stable/43186456. (cited in page 15)

Nevo, Aviv (2001) "Measuring Market Power in the Ready-to-Eat Cereal Industry," *Econometrica*, 69 (2), 307–342, https://doi.org/10.1111/1468-0262.00194. (cited in page 15)

Olley, G. Steven and Ariel Pakes (1996) "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, 64 (6), 1263–1297, http://www.jstor.org/stable/2171831. (cited in page 5, 6, 8, 13, A6)

Pozzi, Andrea and Fabiano Schivardi (2016) "Demand or productivity: what determines firm growth?" *The RAND Journal of Economics*, 47 (3), 608–630, 10.1111/1756-2171.12142. (cited in page 7)

Rubens, Michael (2021) "Market Structure, Oligopsony Power, and Productivity," Working Paper. (cited in page 7, 8)

Salgado, Sergio, Fatih Guvenen, and Nicholas Bloom (2020) "Skewed Business Cycles," Working Paper. (cited in page 16)

Seegmiller, Bryan (2021) "Valuing Labor Market Power: The Role of Productivity Advantages," Working Paper. (cited in page 29)

Smeets, Valerie and Frederic Warzynski (2013) "Estimating productivity with multi-product firms, pricing heterogeneity and the role of international trade," *Journal of International Economics*, 90 (2), 237 – 244, https://doi.org/10.1016/j.jinteco.2013.01.003. (cited in page 7)

Sockin, Jason (2022) "Show Me the Amenity: Are Higher-Paying Firms Better All Around?", Working Paper. (cited in page 16)

Sonnadara, Kanishka (2013) "Audrey Hepburn and the rise of the Vespa," *Motocycle.com*, https://www.motorcycle.com/how-to/audrey-hepburn-and-the-rise-of-the-vespa. (cited in page 15)

Sorkin, Isaac (2018) "Ranking Firms Using Revealed Preference," *The Quarterly Journal of Economics*, 133 (3), 1331–1393, 10.1093/qje/qjy001. (cited in page 16)

Verhoogen, Eric A. (2008) "Trade, Quality Upgrading, and Wage Inequality in the Mexican Manufacturing Sector," *The Quarterly Journal of Economics*, 123 (2), 489–530, 10.1162/qjec.2008.123.2.489. (cited in page 6)

A Appendix: Computing real output and price indices

Computing constant 2018 prices. Products are identified by a code that combines a code for the product (which coincides with the European PRODCOM codes), a Portuguese-specific code that refines the PRODCOM classification, and a code that indicates variants of the product. I compute an average annual price of each product by taking a weighted average using quantity sold as weights. In the time series of prices, I exclude prices that arise from extraordinary changes defined as percentage increases of at least 300% or percentage decreases of at least 75%.

To compute constant prices, I take the most recent price for each product and adjust it to 2018 prices. I prioritize adjustments based on similar products. For instance, suppose the most recent price data for a particular product is 2016. Then, I look at the price changes between 2016 and 2018 for similar products defined as products with the same initial digits in their PRODCOM code. I take the average of price changes of the products that are closest (most number of overlapping PRODCOM digits) and adjust prices based on that. In the worst case where there are no similar products with prices in the same year, I adjust based on inflation.

This fixed price is fixed not only in the cross-section but also in the time-series dimension. That is, I fix this price for all firms over time. I define real output as the quantities sold using these fixed prices to value them. Then, the price index is the nominal total sales divided by the measured real output.

Table A1: Cross-sectional and time-series heterogeneity of defined quantities

		$t = 0$	$t = 1$
One good	Nominal Y	$P^1_{j0}Q^1_{j0}$	$P^1_{j1}Q^1_{j1}$
	Real Y	$\overline{P}^1_0 Q^1_{j0}$	$\overline{P}^1_0 Q^1_{j1}$
	Price	$\frac{P^1_{j0}}{\overline{P}^1_0}$	$\frac{P^1_{j1}}{\overline{P}^1_0}$
Two goods	Nominal Y	$P^1_{j0}Q^1_{j0} + P^2_{j0}Q^2_{j0}$	$P^1_{j1}Q^1_{j1} + P^2_{j1}Q^2_{j1}$
	Real Y	$\overline{P}^1_0 Q^1_{j0} + \overline{P}^2_0 Q^2_{j0}$	$\overline{P}^1_0 Q^1_{j1} + \overline{P}^2_0 Q^2_{j1}$
	Price	$\frac{P^1_{j0}}{\overline{P}^1_0}\frac{\overline{P}^1_0 Q^1_{j0}}{\overline{P}^1_0 Q^1_{j0}+\overline{P}^2_0 Q^2_{j0}} + \frac{P^2_{j0}}{\overline{P}^2_0}\frac{\overline{P}^2_0 Q^2_{j0}}{\overline{P}^1_0 Q^1_{j0}+\overline{P}^2_0 Q^2_{j0}}$	$\frac{P^1_{j1}}{\overline{P}^1_0}\frac{\overline{P}^1_0 Q^1_{j1}}{\overline{P}^1_0 Q^1_{j1}+\overline{P}^2_0 Q^2_{j1}} + \frac{P^2_{j1}}{\overline{P}^2_0}\frac{\overline{P}^2_0 Q^2_{j1}}{\overline{P}^1_0 Q^1_{j1}+\overline{P}^2_0 Q^2_{j1}}$

Discussion. This particular method I propose preserves both the cross-sectional and time-series variation in firm behavior. I illustrate this in Appendix Table A1 by considering a simple two-period economy with only one or two goods. Focusing on the case with one good, what we would observe in the data is "Nominal Y" which is the firm-specific total revenue which is a product of prices and quantities. I choose a constant price $\overline{P}_0^1$ and use this to value the quantities produced and this is "Real Y". The price index is then the ratio of the nominal output and the real output. Notice that in this simple economy, if we focus on a particular time period, variation in the "Real Y" across firms only comes from differences in quantity produced. On the other hand, if we consider a particular firm and compare "Real Y" over time, variation only comes from time-series differences in the quantities produced by the firm. The same argument can be seen in the illustration with two goods and extends to the realistic case with more goods.

B Appendix: Joint dynamics of firm heterogeneity

Under the Markovian assumptions of the model, to specify the dynamics of the latent firm heterogeneity, it is sufficient to specify the conditional density

$$f(\omega_{jt}, \delta_{jt}, S_{jt} \mid \omega_{jt-1}, \delta_{jt-1}, S_{jt-1}). \tag{B1}$$

As discussed in Section 3, we allow for the shocks to productivity, demand, and labor market advantages to be correlated and so we cannot simply decompose the above conditional density as a product of the individual univariate conditional densities $f(\omega_{jt} \mid \omega_{jt-1})$, $f(\delta_{jt} \mid \delta_{jt-1})$, and $f(S_{jt} \mid S_{jt-1})$. In this appendix, I discuss ways of modeling this dependence and, in particular, motivate the triangular representation in Equations (15)–(17).

For conciseness, I will proceed with the discussion as if there were only two latent variables, productivity (ω_{jt}) and demand advantages (δ_{jt}), but it is conceptually straightforward to extend the arguments for the case with more latent variables. The conditional version of Sklar's theorem with absolutely continuous marginals implies

$$\begin{aligned} f(\omega_{jt}, \delta_{jt} \mid \omega_{jt-1}, \delta_{jt-1}) =& f(\omega_{jt} \mid \omega_{jt-1}, \delta_{jt-1}) \times f(\delta_{jt} \mid \omega_{jt-1}, \delta_{jt-1}) \\ & \times c(F(\omega_{jt}|\omega_{jt-1}, \delta_{jt-1}), F(\delta_{jt}|\omega_{jt-1}, \delta_{jt-1}) \mid \omega_{jt-1}, \delta_{jt-1}) \end{aligned} \tag{B2}$$

where $c(\cdot, \cdot \mid \omega_{jt-1}, \delta_{jt-1})$ is the density of the bivariate conditional copula. Specifically, it tells us that the conditional bivariate density of $(\omega_{jt}, \delta_{jt})$ given $(\omega_{jt-1}, \delta_{jt-1})$ can be decomposed into the univariate conditional densities and a copula that captures the dependence structure between the shocks to productivity and demand advantages. Under the conditional independence assumptions of the model, we have following simplifications:

- $f(\omega_{jt} \mid \omega_{jt-1}, \delta_{jt-1}) = f(\omega_{jt} \mid \omega_{jt-1})$ and $F(\omega_{jt} \mid \omega_{jt-1}, \delta_{jt-1}) = F(\omega_{jt} \mid \omega_{jt-1})$
- $f(\delta_{jt} \mid \omega_{jt-1}, \delta_{jt-1}) = f(\delta_{jt} \mid \delta_{jt-1})$ and $F(\delta_{jt} \mid \omega_{jt-1}, \delta_{jt-1}) = F(\delta_{jt} \mid \delta_{jt-1})$
- $c(F(\omega_{jt}|\omega_{jt-1}, \delta_{jt-1}), F(\delta_{jt}|\omega_{jt-1}, \delta_{jt-1}) \mid \omega_{jt-1}, \delta_{jt-1}) = c(F(\omega_{jt}|\omega_{jt-1}), F(\delta_{jt}|\delta_{jt-1}))$

where the last one follows from the assumption that the distribution of the shocks to productivity and demand advantages are independent of the past realizations.

Thus,

$$f(\omega_{jt}, \delta_{jt} \mid \omega_{jt-1}, \delta_{jt-1}) = f(\omega_{jt} \mid \omega_{jt-1}) \times f(\delta_{jt} \mid \delta_{jt-1}) \times c(F(\omega_{jt}|\omega_{jt-1}), F(\delta_{jt}|\delta_{jt-1})). \tag{B3}$$

To implement this, we would need to specify the three components. As discussed in Section 4.1, the first two could be flexibly specified using conditional quantile functions. Thus, we would simply need to augment the model with a parametric copula. This has a couple of drawbacks. First, we would need to select a particular parametric copula which may limit the dependence structures that could be accommodated. Second, once we have selected a copula, this needs to be estimated and the maximum likelihood step needed to estimate the copula parameters may be computationally costly.

We can alternatively use a different decomposition of the conditional density:

$$f(\omega_{jt}, \delta_{jt} \mid \omega_{jt-1}, \delta_{jt-1}) = f(\delta_{jt} \mid \omega_{jt}, \omega_{jt-1}, \delta_{jt-1}) \times f(\omega_{jt} \mid \omega_{jt-1}, \delta_{jt-1}). \tag{B4}$$

As above, we can simplify the second component, $f(\omega_{jt} \mid \omega_{jt-1}, \delta_{jt-1}) = f(\omega_{jt} \mid \omega_{jt-1})$. After this simplification, the model is still more flexible compared to the model in Equation (B3). In particular, we have not imposed the assumption that the distribution of shocks to productivity and demand advantages are independent of past realization. In such a case,

$$f(\delta_{jt} \mid \omega_{jt}, \omega_{jt-1}, \delta_{jt-1}) = f(\delta_{jt} \mid F(\omega_{jt} \mid \omega_{jt-1}), \omega_{jt-1}, \delta_{jt-1}) = f(\delta_{jt} \mid F(\omega_{jt} \mid \omega_{jt-1}), \delta_{jt-1}) \tag{B5}$$

where the first equality follows without loss of generality. The second equality is a simplification that intutitively tells us that past productivity does not carry any additional information about current demand advantages after we control for past demand advantages and the current productivity shock. If the dependence of shocks depended on past realizations, then we cannot make the same simplification. Thus,

$$f(\omega_{jt}, \delta_{jt} \mid \omega_{jt-1}, \delta_{jt-1}) = f(\delta_{jt} \mid F(\omega_{jt} \mid \omega_{jt-1}), \delta_{jt-1}) \times f(\omega_{jt} \mid \omega_{jt-1}). \tag{B6}$$

The main virtue of considering this particular decomposition is in implementation. In particular, we do not need to specify a particular copula, which becomes even more difficult with more than two variables. As such, we can accommodate arbitrary dependence in the shocks. Moreover, the two components can be easily specified using conditional quantile functions which can be estimated through a series of quantile regressions.

Future work would take advantage of the flexibility of this triangular structure. In particular, by estimating models based on Equation (B4), we may allow for the distribution of shocks to arbitrarily depend on past realizations.

C Appendix: Identification in a simpler model

To provide intuition in the model, I consider a simplified model. The model will be simplified along the following lines:

1. the dynamic processes of productivity, demand, and labor market advantages will be linear $AR(1)$ processes;
2. I remove capital as a relevant state variable; and
3. all structural equations and empirical policy functions are log-linear in their arguments.

Simplifications (1) and (3) are restrictive but we might expect nonlinearities to make identification easier by allowing us to exploit higher-order moments of the model. Simplification (2) is simply for convenience. As capital is a dynamic input, it admits natural instruments which we know from Olley and Pakes (1996), for instance.

Production function and productivity. I begin with a Cobb-Douglas production function that only takes materials and labor as inputs. In logs, the production function is

$$y_{jt} = \beta_m m_{jt} + \beta_\ell \ell_{jt} + \omega_{jt} + \varepsilon^y_{jt}, \tag{C1}$$

and we assume that productivity ω_{jt} follows a linear $AR(1)$ process with autoregressive coefficient ρ_ω. Then,

$$\omega_{jt} = \rho_\omega \omega_{jt-1} + \eta^\omega_{jt}, \tag{C2}$$

where η^ω_{jt} are the idiosyncratic innovations to productivity which we assume to be serially uncorrelated and independent of the idiosyncratic shocks in other parts of the model.

Taking the ρ_ω-quasi-first-difference of Equation (C1) and using what we know about the process of ω_{jt} in Equation (C2), we have

$$y_{jt} = \rho_\omega y_{jt-1} + \beta_m(m_{jt} - \rho_\omega m_{jt-1}) + \beta_\ell(\ell_{jt} - \rho_\omega \ell_{jt-1}) + \underbrace{(\eta^\omega_{jt} + \varepsilon^y_{jt} - \rho_\omega \varepsilon^y_{jt-1})}_{\equiv \tilde{\varepsilon}^y_{jt}}. \tag{C3}$$

The OLS estimates of the above equation will lead to biased estimates because of the endogeneity brought by the dynamic nature of the outcome variable. In particular, we have $\text{Cov}\left(y_{jt-1}, \tilde{\varepsilon}^y_{jt}\right) \neq 0$. Moreover, since materials and labor are determined statically, they are correlated with the contemporaneous innovation in productivity η^{ω}_{jt}. Drawing insights from estimation with sequentially exogenous regressors, the past realizations of the outcome $\{y_{jt-2}, ...\}$ are valid instruments. I explore the conditions under which the past material inputs $\{m_{jt-1}, m_{jt-2}, ...\}$ and labor inputs $\{\ell_{jt-1}, \ell_{jt-2}, ...\}$ are also valid instruments.

Consider linearized versions of Equations (10), (7) and (3), which are the material demand, wage-setting, and labor supply equations, respectively.

$$m_{jt} = h_\ell \ell_{jt} + h_\omega \omega_{jt} + h_\delta \delta_{jt} + h_S S_{jt} + \varepsilon^m_{jt} \tag{C4}$$

$$w_{jt} = \gamma_w w_{jt-1} + \gamma_\omega \omega_{jt} + \gamma_\delta \delta_{jt} + \gamma_S S_{jt} + \varepsilon^w_{jt} \tag{C5}$$

$$\ell_{jt} = \theta_w w_{jt} + S_{jt} + \varepsilon^\ell_{jt} \tag{C6}$$

Moreover, we assume that the unobserved demand advantages (δ_{jt}) and labor market advantages (S_{jt}) follow linear $AR(1)$ processes with autoregressive coefficients ρ_δ and ρ_S, respectively:

$$\delta_{jt} = \rho_\delta \delta_{jt-1} + \eta^\delta_{jt} \tag{C7}$$

$$S_{jt} = \rho_S S_{jt-1} + \eta^S_{jt} \tag{C8}$$

where η^δ_{jt} and η^S_{jt} are the idiosyncratic innovations to the demand advantage and labor market advantage processes, respectively. These shocks are serially uncorrelated and are independent to the other shocks in the model.

The past inputs are functions only of the past latent heterogeneity; therefore, they are uncorrelated to future innovations in productivity, and, consequently, to $\tilde{\varepsilon}^y_{jt}$. The remaining question is whether they are able to induce relevant variation to be a valid instrument. Taking ρ_ω-differences of Equations (C4), (C5), and (C6),

$$\begin{aligned} m_{jt} - \rho_\omega m_{jt-1} = & h_\ell(\ell_{jt} - \rho_\omega \ell_{jt-1}) + h_\omega \eta^\omega_{jt} + h_\delta(\rho_\delta - \rho_\omega)\delta_{jt-1} + \eta^\delta_{jt} \\ & + h_S(\rho_S - \rho_\omega) S_{jt-1} + \eta^S_{jt} + (\varepsilon^m_{jt} - \rho_\omega \varepsilon^m_{jt-1}) \end{aligned} \tag{C9}$$

$$\begin{aligned} w_{jt} - \rho_\omega w_{jt-1} = & \gamma_w(w_{jt-1} - \rho_\omega w_{jt-2}) + \gamma_\omega \eta^\omega_{jt} + \gamma_\delta(\rho_\delta - \rho_\omega)\delta_{jt-1} + \gamma_\delta \eta^\delta_{jt} \\ & + \gamma_S(\rho_S - \rho_\omega) S_{jt-1} + \gamma_S \eta^S_{jt} + (\varepsilon^w_{jt} - \rho_\omega \varepsilon^w_{jt-1}) \end{aligned} \tag{C10}$$

$$\ell_{jt} - \rho_\omega \ell_{jt-1} = \theta_w(w_{jt} - \rho_\omega w_{jt-1}) + (\rho_S - \rho_\omega) S_{jt-1} + \eta^S_{jt} + (\varepsilon^\ell_{jt} - \rho_\omega \varepsilon^\ell_{jt}). \tag{C11}$$

The past materials choice m_{jt-1} is a function of past firm heterogeneity including $(\omega_{jt-1}, \delta_{jt-1}, S_{jt-1})$ as well as past errors including $(\varepsilon^m_{jt}, \varepsilon^w_{jt}, \varepsilon^\ell_{jt})$. Then, its relevance as an instrument in Equation (C3) comes from its correlations to past wages $(w_{jt-1}, w_{jt-2}, ...)$ through adjustment costs and $(\delta_{jt-1}, S_{jt-1}, \varepsilon^m_{jt})$ in Equations (C9) and (C11). Correlations through δ_{jt} and S_{jt} are non-zero only if $\rho_\delta \neq \rho_\omega$ and $\rho_S \neq \rho_\omega$. Additionally, we assume that h_δ and θ_w are non-zero implying that the demand and labor market advantages are relevant to these decisions.[1] Under these conditions, input choices in the more distant past $\{m_{jt-2}, ...\}$ are also valid instruments. When these conditions do not hold, and there are no wage adjustment costs, then the relevance of m_{jt-1} only comes from its correlation to ε^m_{jt-1} which could be optimization error but is in general just an idiosyncratic error component. Moreover, inputs from the more distant past are not relevant instruments. The differences in the persistence of the latent components is thus important in order to use the past inputs as valid instruments *and* not rely solely on unidentified errors for identification. The argument for using $\{\ell_{jt-1}, \ell_{jt-2}, ...\}$ as valid instruments is similar.

Demand function and demand advantages. Now I turn to identification of the output demand side of the model, characterized by three key equations. Consider a linearized version of Equation (2), in logs,

$$y_{jt} = \alpha p_{jt} + \delta_{jt} + \varepsilon^d_{jt} \tag{C12}$$

which is the output demand equation of the firms. As discussed above, the price-setting equation is determined once the other inputs are decided. Consider a linear approximation of the price-setting policy function, Equation (9),

$$p_{jt} = g_\ell \ell_{jt} + g_\omega \omega_{jt} + g_\delta \delta_{jt} + g_S S_{jt} + \varepsilon^p_{jt}. \tag{C13}$$

The third component of the demand side is Equation (C7) which describes the dynamics of demand advantages. Taking the ρ_δ-quasi-first-difference of Equation

[1]This argument echoes the results of Gandhi et al. (2020) who study the nonparametric identification of gross output production functions using control function approaches. They argue that without variation in the demand side of the model or adjustment costs, the gross output production function is not nonparametrically identified.

(C12),

$$y_{jt} = \rho_\delta y_{jt-1} + \alpha(p_{jt} - \rho_\delta p_{jt-1}) + \underbrace{\left(\eta^\delta_{jt} + \varepsilon^d_{jt} - \rho_\delta \varepsilon^d_{jt-1}\right)}_{\equiv \tilde{\varepsilon}^d_{jt}}. \tag{C14}$$

As with the production function, OLS estimates are biased since p_{jt} is a contemporaneous decision and depends on the contemporaneous innovation in the demand advantages, η^δ_{jt}. But for the same reason, and if $\rho_\delta \neq \rho_\omega$, past prices $\{p_{jt-1}, p_{jt-2}, ...\}$ are valid instruments as they are uncorrelated to current innovations in demand advantages or demand shocks, and they correlate to current prices as productivity and demand advantages are persistent. Past realizations of other inputs and wages will also be valid for a similar reason. Moreover, as we know from identification of linear dynamic panel data models, past realizations of the outcome $\{y_{jt-2}, ...\}$ are also valid instruments.[2]

Labor supply equation and labor market advantages. Lastly, I turn to identification of the parameters of the labor side of the model. The three components are the labor supply equation, Equation (C6); the wage-setting equation, Equation (C5); and the dynamics of labor market advantages, Equation (C8). Taking ρ_S-differences of Equations (C6) and (C5),

$$\ell_{jt} = \rho_S \ell_{jt-1} + \theta(w_{jt} - \rho_S w_{jt-1}) + \underbrace{\left(\eta^S_{jt} + \varepsilon^\ell_{jt} - \rho_S \varepsilon^\ell_{jt}\right)}_{\equiv \tilde{\varepsilon}^\ell_{jt}} \tag{C15}$$

$$\begin{aligned} w_{jt} - \rho_S w_{jt-1} = &\gamma_\omega(\rho_\omega - \rho_S)\omega_{jt-1} + \gamma_\omega \eta^\omega_{jt} + \gamma_S(\rho_\delta - \rho_S)\delta_{jt-1} + \gamma_S \eta^S_{jt} \\ &+ \gamma_S \eta^S_{jt} + (\varepsilon^w_{jt} - \rho_S \varepsilon^w_{jt-1}). \end{aligned} \tag{C16}$$

Again, we know that OLS estimates of Equation (C15) are biased. As in the previous arguments, past realizations of the outcome $\{\ell_{jt-2}, ...\}$ are valid instruments. With $\rho_\omega \neq \rho_S$ and $\rho_\delta \neq \rho_S$, we have access to additional valid instruments. First, as wages are also a static decision, then past wages $\{w_{jt-1}, w_{jt-2}, ...\}$ are also valid instruments.[3]

[2]With independence of the innovation to productivity, η^ω_{jt}, and the innovation to demand advantages, η^δ_{jt}, as well as $\rho_\delta \neq \rho_\omega$, then $\tilde{\varepsilon}^y_{jt}$ is also a valid instrument.

[3]Moreover, with the additional assumption that the innovations to labor supply advantages are independent of η^ω_{jt} and η^δ_{jt}, then we have additional overidentifying restrictions because $\tilde{\varepsilon}^y_{jt}$ and $\tilde{\varepsilon}^d_{jt}$ are also valid instruments.

Shock variances: covariance structures. The persistence of productivity, demand, and labor market advantages are identified in the arguments above. Thus, the only things left to obtain is the variance-covariance matrix of the shocks. The errors in Equations (C3), (C14), and (C15) are of the form

$$\tilde{\varepsilon}^z_{jt} = \eta^{z'}_{jt} + \varepsilon^z_{jt} - \rho_{z'}\varepsilon^z_{jt-1} \tag{C17}$$

where $(z, z') \in \{(y, \omega), (d, \delta), (\ell, S)\}$. We can show that $\text{Var}(\varepsilon^z_{jt})$ and $\text{Var}(\eta^{z'}_{jt})$ are identified by the covariance structure of $\tilde{\varepsilon}^z_{jt}$ along with the uncorrelatedness assumptions we have previously made about the innovations and measurement error terms. In particular,

$$\text{Var}(\varepsilon^z_{jt}) = -\frac{\text{Cov}(\tilde{\varepsilon}^z_{jt}, \tilde{\varepsilon}^z_{jt-1})}{\rho_{z'}} \tag{C18}$$

$$\text{Var}(\eta^{z'}_{jt}) = \text{Var}(\tilde{\varepsilon}^z_{jt}) - (1 + \rho^2_{z'})\text{Var}(\varepsilon^z_{jt}) \tag{C19}$$

which relies on $\rho_{z'} \neq 0$. Furthermore, we assume that the variances are constant for all t but with more time periods this can easily be relaxed.

The covariances of the shocks to productivity, demand, and labor market advantages are easily obtained from covariances between $\tilde{\varepsilon}^y_{jt}, \tilde{\varepsilon}^d_{jt}$, and $\tilde{\varepsilon}^\ell_{jt}$.

Wage-setting and pass-through. Restating Equation (C10) here, we have

$$\begin{aligned} w_{jt} - \rho_\omega w_{jt-1} =& \gamma_w(w_{jt-1} - \rho_\omega w_{jt-2}) + \gamma_\omega \eta^\omega_{jt} + \gamma_\delta(\rho_\delta - \rho_\omega)\delta_{jt-1} + \gamma_\delta \eta^\delta_{jt} \\ &+ \gamma_S(\rho_S - \rho_\omega)S_{jt-1} + \gamma_S \eta^S_{jt} + (\varepsilon^w_{jt} - \rho_\omega \varepsilon^w_{jt-1}). \end{aligned} \tag{C20}$$

Since we have identified ρ_ω, we can take a regression of $(w_{jt} - \rho_\omega w_{jt-1})$ on $(w_{jt-1} - \rho_\omega w_{jt-2})$ but that gives a biased estimate of γ_w. However, we can instead instrument $(w_{jt-1} - \rho_\omega w_{jt-2})$ with $\tilde{\varepsilon}^y_{jt-1}$ in the same regression to get γ_w. It is straightforward so see that it is a relevant instrument. The exclusion restriction arises from the independence of the idiosyncratic errors of productivity ε^y_{jt}, and that the shock to productivity η^ω_{jt-1} is independent of firm heterogeneity at $t-1$ and is also independent of the future shock at t. We can obtain overidentifying restrictions as $\tilde{\varepsilon}^d_{jt}$ and $\tilde{\varepsilon}^\ell_{jt}$ are valid instruments in the ρ_δ-quasi-first-differenced and ρ_s-quasi-first-differenced versions of the wage equation, respectively.

With γ_w, we can write

$$w_{jt} - \gamma_w w_{jt-1} = \gamma_\omega \omega_{jt} + \gamma_\delta \delta_{jt} + \gamma_S S_{jt} + \varepsilon^w_{jt}. \tag{C21}$$

Then,

$$\gamma_\omega = \frac{\mathbb{E}((w_{jt} - \gamma_w w_{jt-1})\widetilde{\varepsilon}^y_{jt})}{\text{Var}(\eta^\omega_{jt})}, \ \gamma_\delta = \frac{\mathbb{E}((w_{jt} - \gamma_w w_{jt-1})\widetilde{\varepsilon}^d_{jt})}{\text{Var}(\eta^\delta_{jt})}, \ \gamma_S = \frac{\mathbb{E}((w_{jt} - \gamma_w w_{jt-1})\widetilde{\varepsilon}^\ell_{jt})}{\text{Var}(\eta^S_{jt})} \tag{C22}$$

which uses the variances which have previously been shown to be identified. This result is important as it shows the identification of the pass-through parameters of interest.

Material input. Restating Equation (C4), we have

$$m_{jt} = h_\ell \ell_{jt} + h_\omega \omega_{jt} + h_\delta \delta_{jt} + h_S S_{jt} + \varepsilon^m_{jt}. \tag{C23}$$

Identifying the parameters of this equation is similar to how we obtained the parameters in the wage-setting equation. We can use past $\widetilde{\varepsilon}^y_{jt-1}, \widetilde{\varepsilon}^d_{jt-1}$, and $\widetilde{\varepsilon}^\ell_{jt-1}$ in quasi-first-differenced versions of the materials equations to obtain h_ℓ. Once we have h_ℓ, then we can use contemporaneous correlations with $\widetilde{\varepsilon}^y_{jt}, \widetilde{\varepsilon}^d_{jt}$, and $\widetilde{\varepsilon}^\ell_{jt}$ to obtain the pass-through parameters.

D Appendix: Estimation of the output demand elasticity

We are interested in obtaining an estimate of α_p in the output demand equation

$$y_{jt} = \alpha_p p_{jt} + \delta_{jt} + \varepsilon^d_{jt}, \tag{D1}$$

under some specification of the process of the demand advantages, Equation (5). We will use this estimate in a second-stage where we estimate the rest of the parameters of the model. This two-stage estimator may not be as efficient as estimators that jointly estimate all the parameters but it may be computationally more stable.

I will construct a GMM-based estimator of α_p. To be more transparent, for this argument, I first consider a very simple specification of the dynamics of demand advantages, based on a specification of the conditional quantile functions,

$$\delta_{jt} = \rho_1(v^\delta_{jt})\delta_{jt-1} + \rho_2(v^\delta_{jt})\delta^2_{jt-1}, \tag{D2}$$

where v^δ_{jt} is a uniformly distributed random variable that is serially uncorrelated, in fact, statistically independent of past shocks, outcomes and choices of the firm. Then, since $\delta_{jt} = y_{jt} - \alpha_p p_{jt} - \varepsilon^d_{jt}$,

$$\begin{aligned} \xi_{jt} \equiv & (y_{jt} - \alpha_p p_{jt}) - \rho_1(v^\delta_{jt})(y_{jt-1} - \alpha_p p_{jt-1}) - \rho_2(v^\delta_{jt})(y_{jt-1} - \alpha_p p_{jt-1})^2 - \overline{\rho}_0 \\ = & \varepsilon^d_{jt} - \rho_1(v^\delta_{jt})\varepsilon^d_{jt-1} + \rho_2(v^\delta_{jt})(\varepsilon^d_{jt-1})^2 - \rho_2(v^\delta_{jt})(y_{jt-1} - \alpha_p p_{jt-1})\varepsilon^d_{jt-1} - \overline{\rho}_0 \end{aligned}. \tag{D3}$$

We find conditional moments of the form $\mathbb{E}(\xi_{jt} \mid \Omega_{jt}) = 0$. Because of the strong conditional independence assumptions of the model, we can include past outcomes and choices in Ω_{jt}. In particular, $\Omega_{jt} = \{y_{t-2}, y_{t-3}, ..., p_{t-2}, p_{t-3}, ..., k_{jt-1}, k_{jt-2}, ...\}$. Focusing on the most RHS expression, the terms with ε^d_{jt} and ε^d_{jt-1} have conditional expectation given Ω_{jt}. However, $\mathbb{E}(\rho_2(v^\delta_{jt})(\varepsilon^d_{jt-1})^2 \mid \Omega_{jt}) = \overline{\rho}_2\sigma^2_d$, with $\overline{\rho}_2 = \mathbb{E}(\rho_2(v^\delta_{jt}))$, which is possibly non-zero but is constant not dependent on elements of Ω_{jt}. Thus, including the extra parameter $\overline{\rho}_0$ in our definition of ξ_{jt} is convenient to take into account these non-zero but constant elements and obtain zero conditional mean of ξ_{jt}. Note that we needed the stronger conditional independence assumptions to get independence of higher order moments of ε^d_{jt-1} from the variables in Ω_{jt}.

Now, we have conditional moments

$$\mathbb{E}\left[(y_{jt}-\alpha_p p_{jt})-\rho_1(v^{\delta}_{jt})(y_{jt-1}-\alpha_p p_{jt-1})-\rho_2(v^{\delta}_{jt})(y_{jt-1}-\alpha_p p_{jt-1})^2-\overline{\rho}_0 \mid \Omega_{jt}\right]=0 \tag{D4}$$

but we do not observe v^{δ}_{jt}. In fact, we are not interested in learning the specific functions $\rho_1(\cdot)$ or $\rho_2(\cdot)$ as these are objects which we can estimate the second-stage. As such, we can just define "nuisance" parameters $\overline{\rho}_1 = \mathbb{E}(\rho_1(v^{d}_{jt}))$ and $\overline{\rho}_2 = \mathbb{E}(\rho_2(v^{d}_{jt}))$, and write

$$\mathbb{E}\left[\underbrace{(y_{jt}-\alpha_p p_{jt})-\overline{\rho}_1(y_{jt-1}-\alpha_p p_{jt-1})-\overline{\rho}_2(y_{jt-1}-\alpha_p p_{jt-1})^2-\overline{\rho}_0}_{\equiv\overline{\overline{\xi}}_{jt}(D_{jt};\alpha_p,\overline{\rho})} \mid \Omega_{jt}\right]=0 \tag{D5}$$

with data D_{jt}, which are conditional moment conditions that we can now operationalize in a GMM estimator. Specifically, consider a vector of instruments $Z_{jt}=(z_1(\Omega_{jt}),...,z_N(\Omega_{jt}))$, we consider an estimator which roughly resembles

$$(\widehat{\alpha}_p,\widehat{\overline{\rho}}) = \underset{\alpha,\overline{\rho}}{\operatorname{argmin}} \left(\frac{1}{\overline{\overline{N}}}\sum_{j,t} Z'_{jt}\overline{\overline{\xi}}_{jt}(D_{jt};\alpha_p,\overline{\rho})\right)' W \left(\frac{1}{\overline{\overline{N}}}\sum_{j,t} Z'_{jt}\overline{\overline{\xi}}_{jt}(D_{jt};\alpha_p,\overline{\rho})\right), \tag{D6}$$

for some weighting matrix W.

In practice, I consider the parametric specification of the dynamics of demand advantages given in Equation (19). I use the instruments

$$Z_{jt}=(y_{jt-2},y_{jt-3},p_{jt-2},p_{jt-3},k_{jt-1},k_{jt-2}) \tag{D7}$$

and an identity weighting matrix.[4]

[4]The instrument set may also be augmented with higher-order polynomials of past outcomes and choices to possibly improve efficiency of the estimator. There are alternative weighting matrices that could be considered.

E Appendix: Second-stage estimation details

To simplify notation, in this section I assume that we observe a balanced panel of J firms for T periods. For each firm, we observe $D_j = \{y_j^T, m_j^T, \ell_j^T, k_j^T, w_j^T, p_j^T\}$ where I use the shorthand z_i^T to denote $\{z_{j1}, z_{j2}, ..., z_{jT}\}$. Productivity ($\omega_j^T$), demand advantages ($\delta_j^T$), and labor market advantages (S_j^T) are unobserved. We are interested in obtaining estimates for the parameters that govern the dynamics of latent heterogeneity $\gamma = (a_k^\omega, a_k^\delta, a_k^S)$, and the parameters that describe the technology, environment, initial conditions, and response functions of the firm $\mu = (\beta, \alpha, \theta, b_k^m, b_k^w, b_k^k, b_k^{w_1}, b_k^{\omega_1}, b_k^{\delta_1}, b_k^{S_1}, \sigma)$.

E.1 Likelihood function

The individual complete-data likelihood is

$$\begin{aligned}
f(D_j, \omega_j^T, \delta_j^T, S_j | k_{j1}) = & \prod_{t=1}^{T} f(y_{jt}, m_{jt}, \ell_{jt}, p_{jt} \mid k_{jt}, w_{jt}, \omega_{jt}, \delta_{jt}, S_{jt}) \\
& \times \prod_{t=2}^{T} f(w_{jt} \mid w_{jt-1}, k_{jt}, \omega_{jt}, \delta_{jt}, S_{jt}) \\
& \times \prod_{t=2}^{T} f(k_{jt} \mid k_{jt-1}, w_{jt-1}, \omega_{jt-1}, \delta_{jt-1}, S_{jt}) \\
& \times \prod_{t=2}^{T} f(\omega_{jt}, \delta_{jt}, S_{jt} \mid \omega_{jt-1}, \delta_{jt-1}, S_{jt}) \\
& \times f(w_{j1} \mid k_{j1}, \omega_{j1}, \delta_{j1}, S_{j1}) \times f(\omega_{j1}, \delta_{j1}, S_{j1} | k_{j1}).
\end{aligned} \tag{E1}$$

Simultaneity. Output y is simultaneously determined by production and demand, Equations (1) and (2), respectively. However, this specific model can be shown to have a triangular structure.[5] Moreover, since the individual errors of Equations (12), (13), (14), and (26) are assumed distributed Normal and are inde-

[5]In the general case, the likelihood of the simultaneous system with Gaussian errors will include a non-trivial term related to the Jacobian of the system (Amemiya, 1977). With a triangular structure, and appropriate normalization, this term is a constant in the likelihood.

pendent of each other, then

$$f(y_{jt}, m_{jt}, \ell_{jt}, p_{jt} \mid k_{jt}, w_{jt}, \omega_{jt}, \delta_{jt}, S_{jt}) \propto \left[\frac{1}{\sigma_d}\phi\left(\frac{y_{jt} - Q^d(p_{jt}, \delta_{jt})}{\sigma_d}\right)\right] \times \left[\frac{1}{\sigma_y}\phi\left(\frac{y_{jt} - Q^y(k_{jt}, m_{jt}, \ell_{jt}, \omega_{jt})}{\sigma_y}\right)\right] \times \left[\frac{1}{\sigma_\ell}\phi\left(\frac{\ell_{jt} - Q^\ell(w_{jt}, S_{jt})}{\sigma_\ell}\right)\right] \times \left[\frac{1}{\sigma_m}\phi\left(\frac{m_{jt} - Q^m(k_{jt}, \ell_{jt}, \omega_{jt}, \delta_{jt}, S_{jt})}{\sigma_m}\right)\right] \tag{E2}$$

Note that if the latent variables were observed, the simultaneity is inconsequential and Equations (12), (13), (14), and (26) could be estimated using OLS.

Conditional density of latent heterogeneity. Equations (18), (19), and (20) are flexible parmetrizations of the conditional quantile functions of the latent firm heterogeneity. However, the parameters a_k^ω, a_k^δ, and a_k^S involve a continuum of parameters indexed by $\tau \in (0,1)$ which makes computing the likelihood infeasible. The insight of Arellano and Bonhomme (2016) is that we can instead use a particular approximation of the quantile functions that admits a simple closed-form approximation of the conditional density.

To illustrate, consider the conditional distribution of ω_{jt} conditional on ω_{jt-1} whose conditional quantiles are specified in Equation (18). We approximate $a_k^\omega(\tau)$ as piecewise-linear interpolating splines on the grid $[\tau_1, \tau_2]$, $[\tau_2, \tau_3]$, ..., $[\tau_{L-1}, \tau_L]$ contained in the unit interval. Furthermore, we extend the intercepts $a_0^\omega(\tau)$ such that they are the quantiles of an exponential distribution on $(0, \tau_1]$ and $[\tau_L, 1)$ with parameters λ_-^ω and λ_+^ω, respectively. Denoting $a_{k\ell}^\omega = a_k^\omega(\tau_\ell)$, then a_k^ω depends on $\{a_{k1}^\omega, ..., a_{kL}^\omega, \lambda_-^\omega, \lambda_+^\omega\}$. Then, we can write

$$\begin{aligned} f(\omega_{jt} \mid \omega_{jt-1}) = & \mathbb{1}\{\omega_{jt} < A_{jt}^\omega(1)\} \times \tau_1 \lambda_-^\omega \exp\left[\lambda_-^\omega(\omega_{jt} - A_{jt}^\omega(1))\right] \\ & + \sum_{\ell=1}^{L-1} \mathbb{1}\{A_{jt}^\omega(\ell) \le \omega_{jt} < A_{jt}^\omega(\ell+1)\} \times \frac{\tau_{\ell+1} - \tau_\ell}{A_{jt}^\omega(\ell+1) - A_{jt}^\omega(\ell)} \\ & + \mathbb{1}\{\omega_{jt} \ge A_{jt}^\omega(L)\} \times (1 - \tau_L)\lambda_+^\omega \exp\left[-\lambda_+^\omega(\omega_{jt} - A_{jt}^\omega(L))\right] \end{aligned} \tag{E3}$$

where $A^{\omega}_{jt}(\ell) = \sum_{k=0}^{K} a^{\omega}_{k\ell}\varphi^{\omega}_{k}(\omega_{jt-1})$ for all (j,t,ℓ). This is similarly done for the conditional densities of the other two latent variables.

E.2 Stochastic EM

I adapt a similar estimation algorithm as in Arellano and Bonhomme (2016) and Arellano et al. (2017) based on the stochastic EM algorithm. The stochastic EM is a simulated version of the classical EM algorithm developed by Dempster et al. (1977). A main difference from typical EM algorithms is that the M-step is not based on likelihood maximization. Instead, we implement the M-step as a series of quantile regressions and linear regressions which makes the step more computationally efficient.

We start with an initial guess of the parameters $(\gamma^{(0)}, \mu^{(0)})$. For $m = 1, ..., M$, we alternate between the following two steps:

1. *Stochastic E-step.* Take S draws of the latent variables from the conditional distribution. That is, draw $\{\omega^{(s)}_{jt}, \delta^{(s)}_{jt}, S^{(s)}_{jt}\}_{t=1}^{T}$, for $s = 1, ..., S$, from

$$f\left(\omega^{T}_{j}, \delta^{T}_{j}, S^{T}_{j} \mid D_j; \gamma^{(m-1)}, \mu^{(m-1)}\right). \tag{E4}$$

   In practice, this is done through a random-walk Metropolis-Hastings MCMC algorithm detailed below.

2. *M-step.* As mentioned, I obtain $(\gamma^{(m)}, \mu^{(m)})$ in a series of quantile regressions and OLS regressions. Specifically,

   - Under the chosen parametrization, $a^{z}_{k}(\tau)$ is fully specified by the parameters $\{a^{z}_{k1}, ..., a^{z}_{kL}, \lambda^{z}_{-}, \lambda^{z}_{+}\}$. This is true for each of the elements of $\gamma = (a^{\omega}_{k}, a^{\omega_1}_{k}, a^{\delta}_{k}, a^{\delta_1}_{k}, a^{S}_{k}, a^{S_1}_{k})$. To illustrate, the parameters of dynamics of the latent variables satisfy, for $\ell = 1, ..., L$,

$$\{\widehat{a}^{z,(m)}_{0\ell}, ..., \widehat{a}^{z,(m)}_{K\ell}\} = \underset{a^{z}_{0\ell},...,a^{z}_{K\ell}}{\operatorname{argmin}} \sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=2}^{T} \rho_{\tau_\ell}\left(z^{(s)}_{it} - \sum_{k=0}^{K} a^{z}_{k\ell}\varphi_k\left(z^{(s)}_{jt-1}, age_{it}\right)\right) \tag{E5}$$

     where $\rho_\tau(u) = u(\tau - \mathbb{1}\{u \leq 0\})$ is the so-called "check function". This optimization problem corresponds to a standard quantile regression.

Moreover,

$$\widehat{\lambda}_{-}^{z,(m)} = -\frac{\sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=2}^{T} \mathbb{1}\left\{z_{it}^{(s)} \leq \widehat{A}_{jts}^{z,(m)}(1)\right\}}{\sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=2}^{T}\left(z_{it}^{(s)} - \widehat{A}_{jts}^{z,(m)}(1)\right)\mathbb{1}\left\{z_{it}^{(s)} \leq \widehat{A}_{jts}^{z,(m)}(1)\right\}} \tag{E6}$$

and

$$\widehat{\lambda}_{+}^{z,(m)} = \frac{\sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=2}^{T} \mathbb{1}\left\{z_{it}^{(s)} \geq \widehat{A}_{jts}^{z,(m)}(L)\right\}}{\sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=2}^{T}\left(z_{it}^{(s)} - \widehat{A}_{jts}^{z,(m)}(L)\right)\mathbb{1}\left\{z_{it}^{(s)} \geq \widehat{A}_{jts}^{z,(m)}(L)\right\}} \tag{E7}$$

where $\widehat{A}_{jts}^{z,(m)}(\ell) = \sum_{k=0}^{K} \widehat{a}_{k\ell}^{z,(m)} \varphi_k\left(z_{jt-1}^{(s)}, age_{jt}\right)$ for all (j, t, ℓ, s).

- The parameters of the production function $(\beta^{(m)})$, demand function $(\alpha^{(m)})$, labor supply function $(\theta^{(m)})$, response functions $\left(b_k^{m,(m)}, b_k^{w,(m)}, b_k^{i,(m)}\right)$, and initial conditions $\left(b_k^{w_1,(m)}, b_k^{\omega_1,(m)}, b_k^{\delta_1,(m)}, b_k^{S_1,(m)}\right)$ satisfy least squares criteria. For instance, for the materials response function,

$$\{\widehat{b}_0^{m,(m)}, ..., \widehat{b}_K^{m,(m)}\} = \underset{b_0,...,b_K}{\operatorname{argmin}} \sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=1}^{T}\left(m_{jt} - \sum_{k=0}^{K} b_k \varphi_k^m\left(k_{jt}, \ell_{jt}, \omega_{jt}^{(s)}, \delta_{jt}^{(s)}, S_{jt}^{(s)}\right)\right)^2, \tag{E8}$$

and the corresponding variance of the equation residual is estimated as

$$\sigma_m^{2,(m)} = \frac{1}{SJT}\sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=1}^{T}\left(m_{jt} - \sum_{k=0}^{K} \widehat{b}_k^{m,(m)} \varphi_k^m\left(k_{jt}, \ell_{jt}, \omega_{jt}^{(s)}, \delta_{jt}^{(s)}, S_{jt}^{(s)}\right)\right)^2. \tag{E9}$$

The algorithm is stopped after M iterations and the parameter estimates is an average of the last few parameter draws; that is, $(\widehat{\gamma}, \widehat{\mu}) = \frac{1}{\widetilde{M}} \sum_{m=M-\widetilde{M}-1}^{M} \left(\gamma^{(m)}, \mu^{(m)}\right)$. In practice, I run the algorithm for $M = 150$ iterations and take the mean of the last $\widetilde{M} = 30$ iterations.

Drawing from the posterior. The conditional distribution is proportional to the likelihood. As we can compute the likelihood, Markov Chain Monte Carlo (MCMC) methods is an appealing method to draw from the conditional distribution. In this paper, I follow Arellano et al. (2017) and use a random-walk Metropolis-Hastings algorithm.[6] To ease notation, let $\eta_j = (\omega_j^T, \delta_j^T, S_j^T)$, and D_j be all the data we observe from firm j. We start with an initial draw $\eta_j^{(0)}$, and for $i = 1, ..., N$,

1. Proposal: $\eta_j^* = \eta_j^{(i-1)} + \varepsilon_j$ with $\varepsilon_j \sim \mathcal{N}(0, \Sigma)$
2. Compute the acceptance probability

$$\rho(\eta_j^* \mid \eta^{(i-1)}) = \min\left\{1, \frac{f\left(D_j \mid \eta_j^*\right)}{f\left(D_j \mid \eta_j^{(i-1)}\right)}\right\} \tag{E10}$$

3. Draw u from $\mathcal{U}(0,1)$ and we assign the new draw as follows

$$\eta_j^{(i)} = \begin{cases} \eta_j^* & \text{if } u \leq \rho(\eta_j^* \mid \eta^{(i-1)}) \\ \eta_j^{(i-1)} & \text{otherwise.} \end{cases} \tag{E11}$$

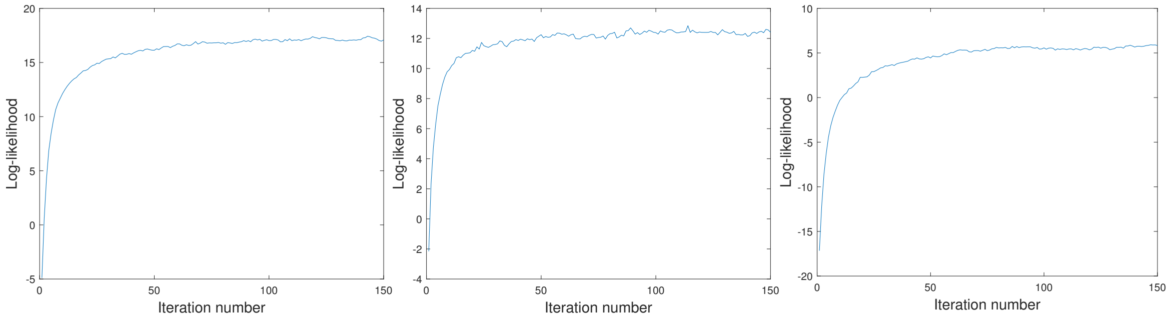
This produces a chain of draws from the conditional distribution of interest. In practice, I implement a block-type of MH algorithm. I set $N = 100$. The last $S < N$ draws are used in the subsequent M-step. To ease the burden on computation, I choose $S = 1$. The variance of the random walk proposal is calibrated to target an acceptance rate of around 30%.

[6] There are alternative MCMC methods (e.g., slice sampling) and alternative simulation methods for proposals (e.g., particle filters).

F Appendix: Model fit

Figure F1: Convergence in EM

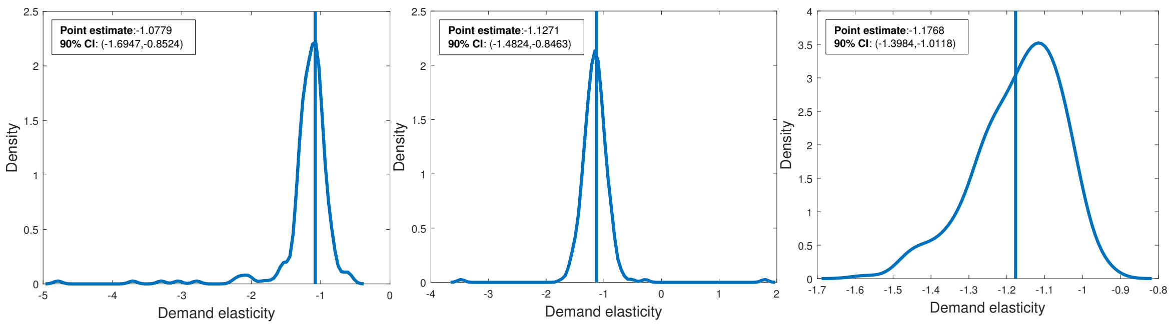
(a) Sector 10 (b) Sector 14 (c) Sector 25



Notes: Path of complete-data log-likelihood over EM iterations in sector-specific estimations. Sector 10 is food products, Sector 14 is clothing, and Sector 25 is metal products.

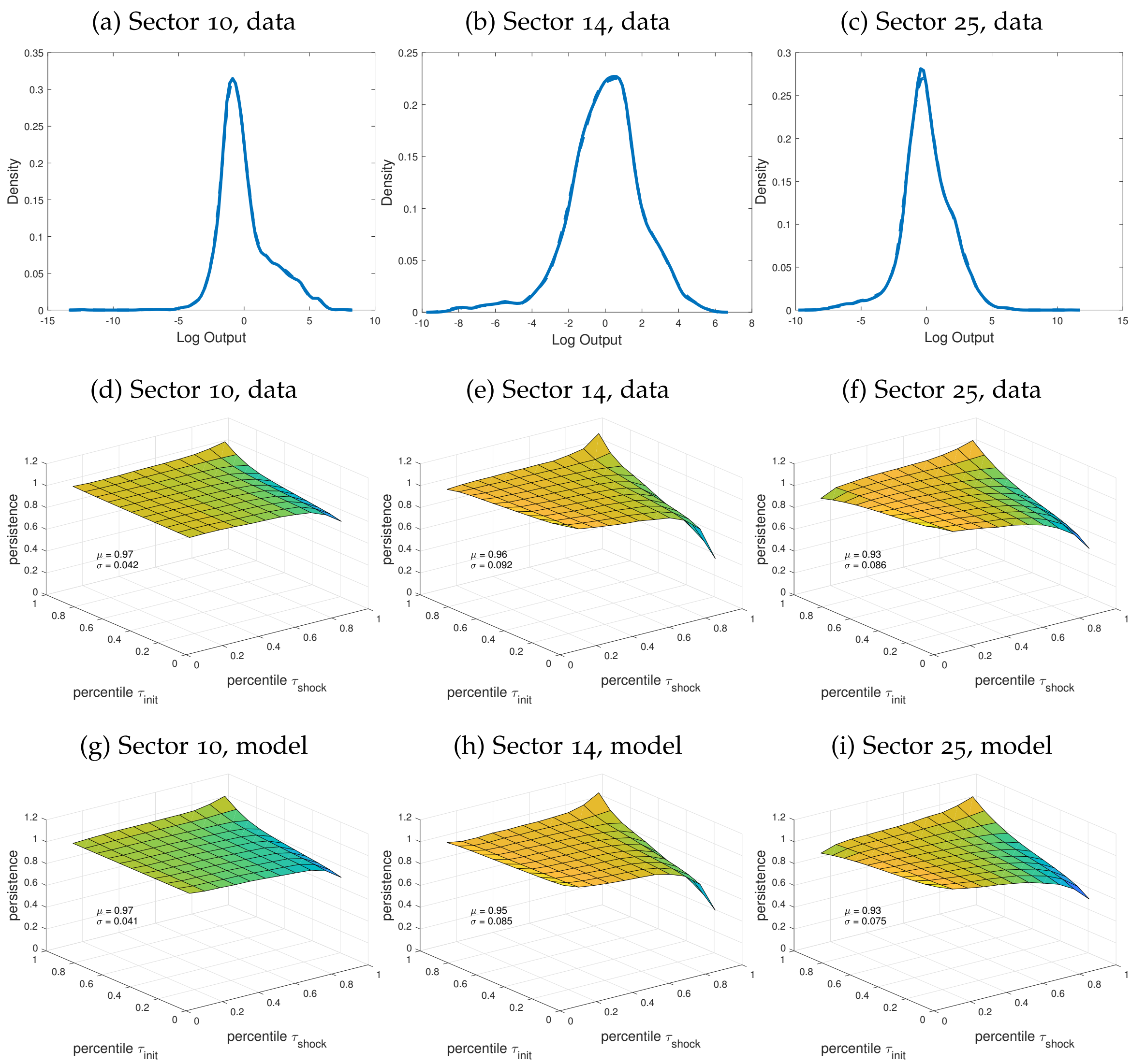
Figure F2: Demand elasticity bootstrap distributions

(a) Sector 10 (b) Sector 14 (c) Sector 25



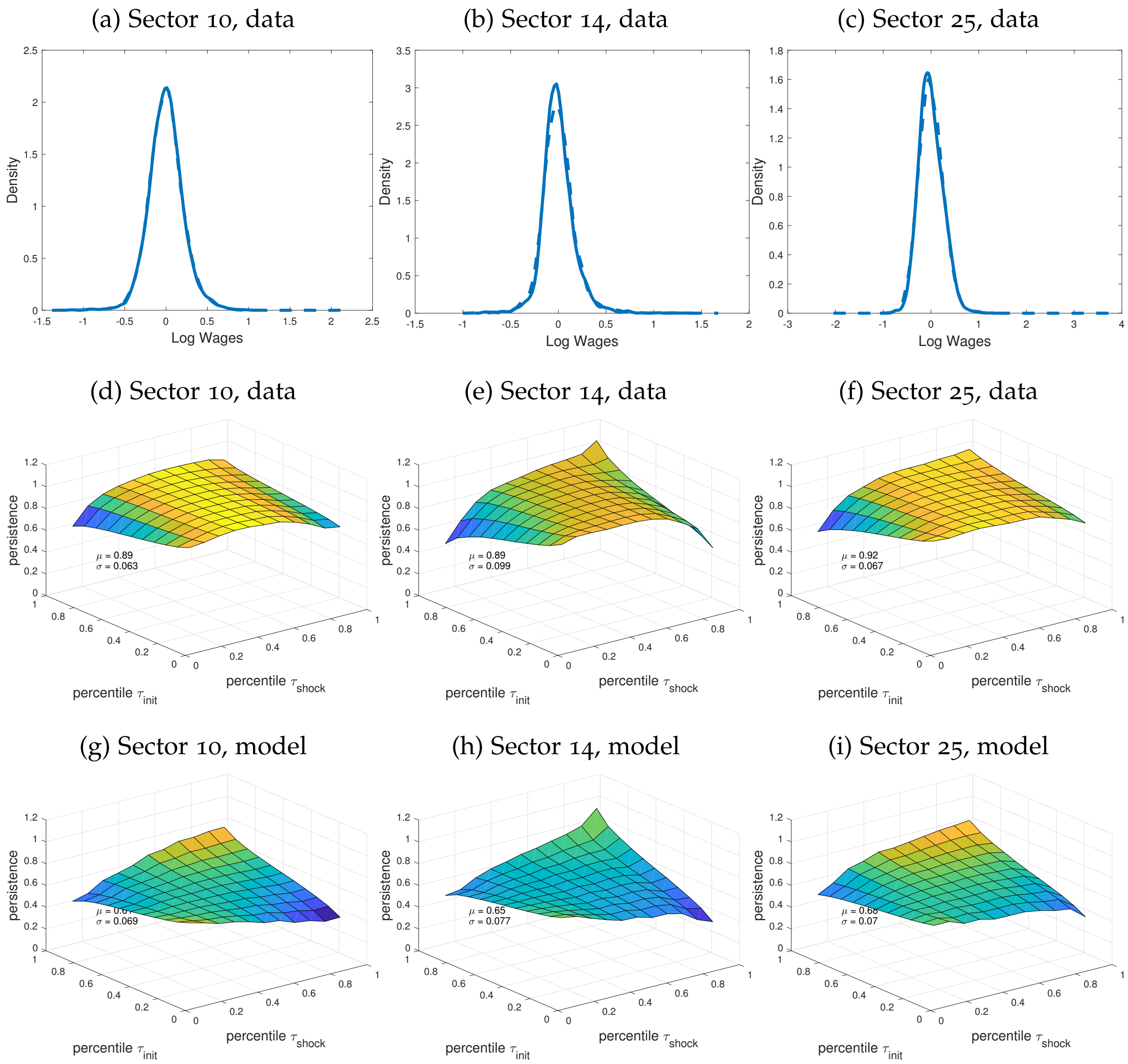
Notes: Bootstrap densities of the demand elasticities clustered at the firm level estimated by GMM (see Appendix D for details) using 250 bootstrap replications. Point estimate (vertical line) and 90% confidence interval also reported. Sector 10 is food products, Sector 14 is clothing, and Sector 25 is metal products.

Figure F3: Fit on output



Panels (a)–(c) present the sector-specific marginal distributions of output in the data (solid) and in simulation (dotted). Panels (d)–(f) show the persistence of output in the data while Panels (g)–(i) show the persistence of output in simulation. Persistence is measured as the average derivative of the conditional quantile of y_{jt} *on* y_{jt-1} *with respect to* y_{jt-1}*. Materials, capital, and labor are fixed in the simulation. Sector 10 is food products, Sector 14 is clothing, and Sector 25 is metal products.*

Figure F4: Fit on wages



Panels (a)–(c) present the sector-specific marginal distributions of log-wages in the data (solid) and in simulation (dotted). Panels (d)–(f) show the persistence of log-wages in the data while Panels (g)–(i) show the persistence of log-wages in simulation. Persistence is measured as the average derivative of the conditional quantile of $\ln w_{jt}$ *on* $\ln w_{jt-1}$ *with respect to* $\ln w_{jt-1}$. *Capital is taken as fixed in the simulation. Sector 10 is food products, Sector 14 is clothing, and Sector 25 is metal products.*

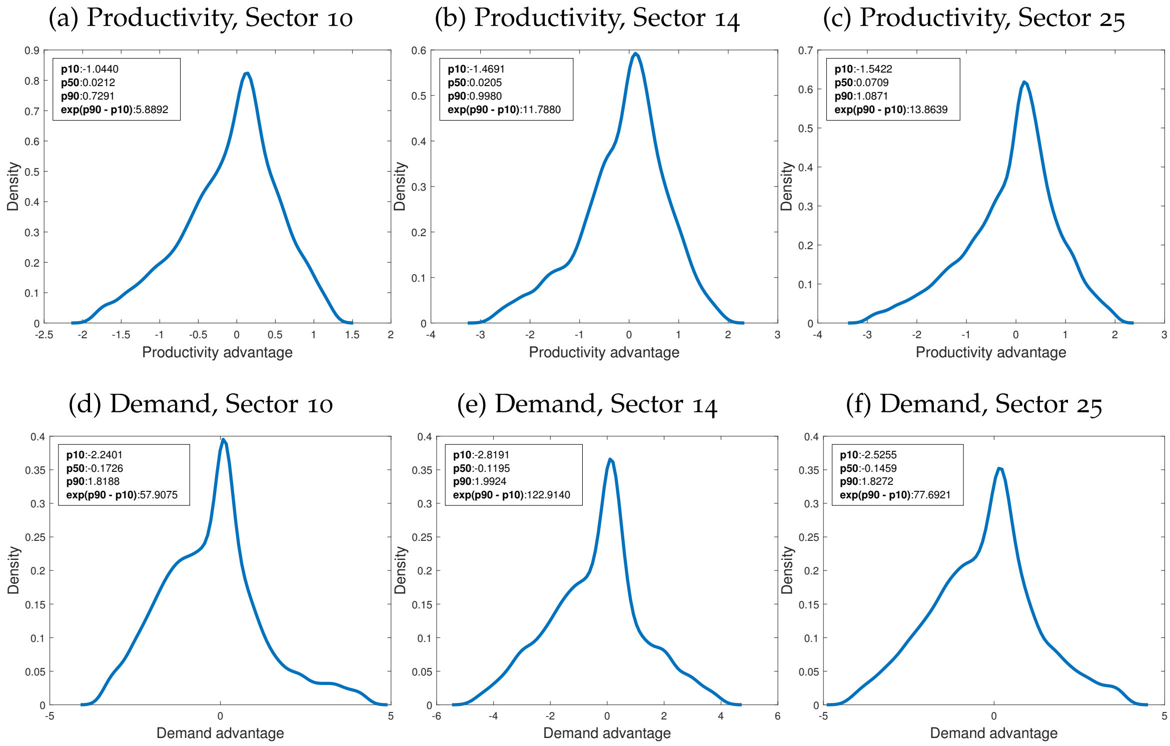
Table F1: Production function estimates

	Production Function			
	β_k	β_ℓ	β_m	$\beta_k + \beta_\ell + \beta_k$
Food products (Sector 10)	0.1338	0.1124	0.7376	0.9838
Clothing (Sector 14)	0.2718	0.3132	0.4138	0.9988
Metal products (Sector 25)	0.2632	0.1560	0.5891	1.0083

Notes: Point estimates of the production function parameters. The sum of the input elasticities in the Cobb-Douglas specification, indicative of returns to scale, also reported.

G Appendix: Additional results

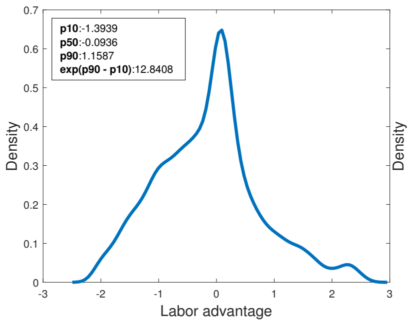
Figure G1: Cross-sectional distribution of productivity and demand advantages, by sector



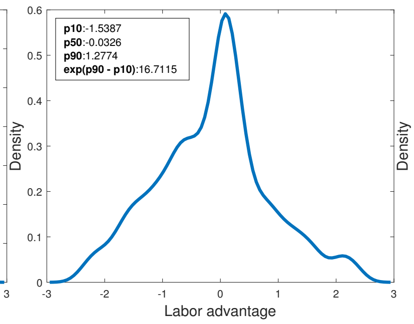
Notes: Panels (a)–(c) and (d)–(f) present sector-specific estimates of the cross-sectional distribution of productivity and demand advantages, respectively. Also reported are the 10th, 50th, and 90th percentiles of the distributions. Dispersion of the distributions measured as $\exp(P90 - P10)$ *also reported. Kernel densities estimated on data with top and bottom 2% trimmed. Percentiles computed with full data. Sector 10 is food products, Sector 14 is clothing, and Sector 25 is metal products.*

Figure G2: Labor market advantages, labor supply elasticity, and implied static markdowns, by sector

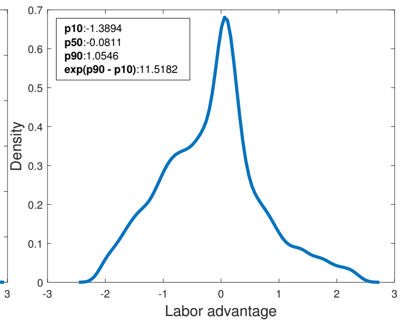
(a) Labor market advantages, Sector 10



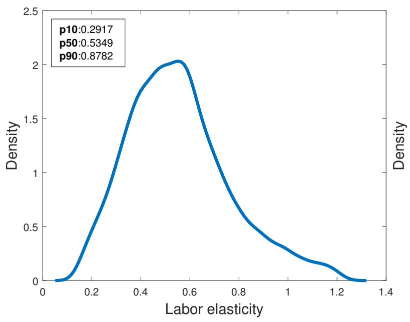
(b) Labor market advantages, Sector 14



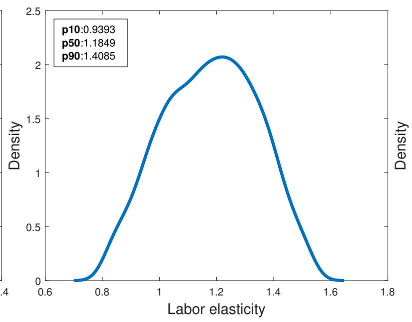
(c) Labor market advantages, Sector 25



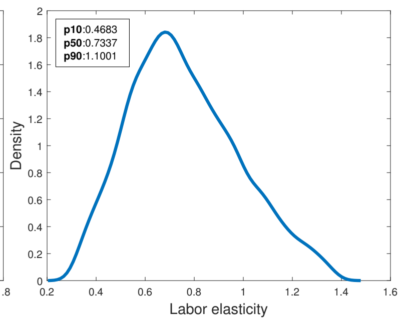
(d) Labor supply elasiticities, Sector 10



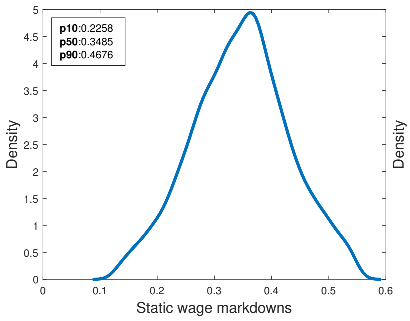
(e) Labor supply elasiticities, Sector 14



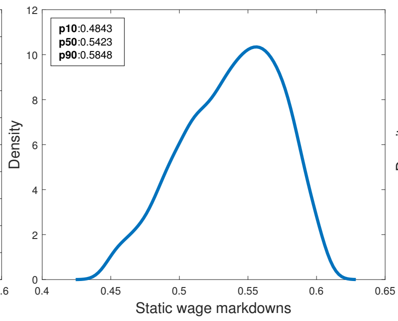
(f) Labor supply elasiticities, Sector 25



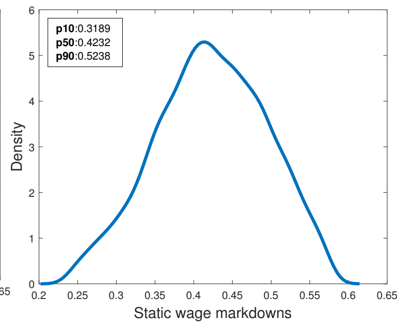
(g) Static markdowns, Sector 10



(h) Static markdowns, Sector 14

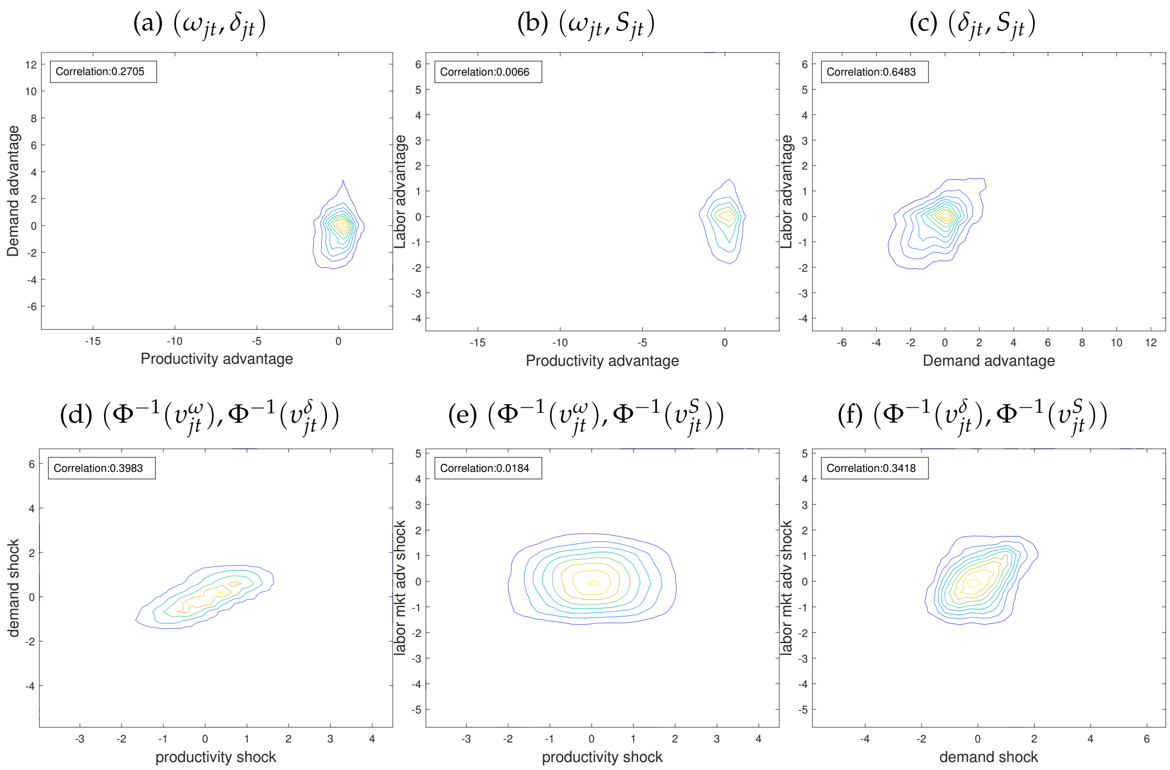


(i) Static markdowns, Sector 25



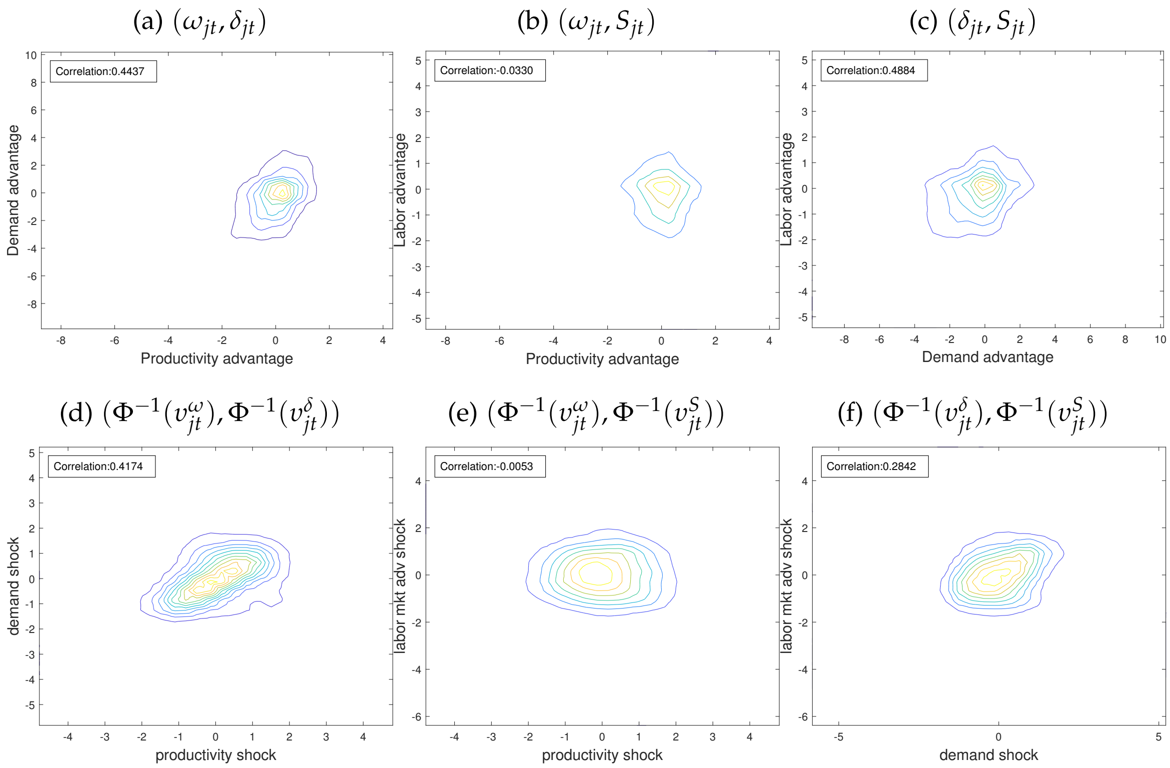
Notes: Panels (a)–(c) present estimates of the distributions of labor market advantages. Panels (d)–(f) present estimates of the labor supply elasticities faced by firms implied by the labor supply function in Equation (14). Panels (g)–(i) present estimates of the distribution of markdowns implied by the labor supply elasticities and static wage-setting, specifically, measured as $\varepsilon^{\ell w}_{jt}/(1+\varepsilon^{\ell w}_{jt})$ *where* $\varepsilon^{\ell w}_{jt}$ *is the labor supply elasticity of wages. Kernel densities estimated on data with top and bottom 2% trimmed. Percentiles computed with full data. Sector 10 is food products, Sector 14 is clothing, and Sector 25 is metal products.*

Figure G3: Joint density of firm heterogeneity and shocks, Sector 10



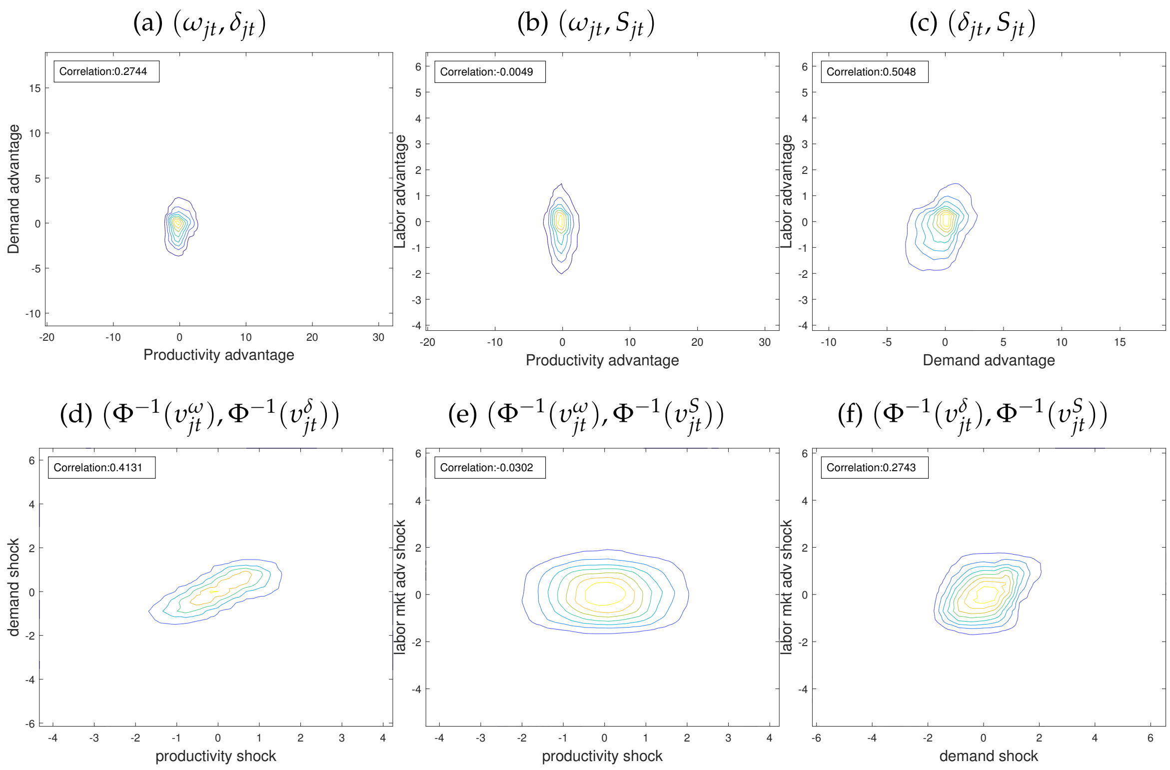
Notes: Sector 10, food products. Panels (a)–(c) are contour plots of the estimated joint distributions of productivity and demand advantages, productivity and labor market advantages, and demand and labor market advantages, respectively. Panels (d)–(f) are contours of the estimated copula densities of the shocks to productivity (v^{ω}_{jt}), demand (v^{δ}_{jt}), and labor market advantages (v^{S}_{jt}). As a graphical convention, I rescale the marginals of the shocks so they are standard normal. Correlations also reported.

Figure G4: Joint density of firm heterogeneity and shocks, Sector 10



Notes: Sector 14, clothing. Panels (a)–(c) are contour plots of the estimated joint distributions of productivity and demand advantages, productivity and labor market advantages, and demand and labor market advantages, respectively. Panels (d)–(f) are contours of the estimated copula densities of the shocks to productivity (v^{ω}_{jt}), demand (v^{δ}_{jt}), and labor market advantages (v^{S}_{jt}). As a graphical convention, I rescale the marginals of the shocks so they are standard normal. Correlations also reported.

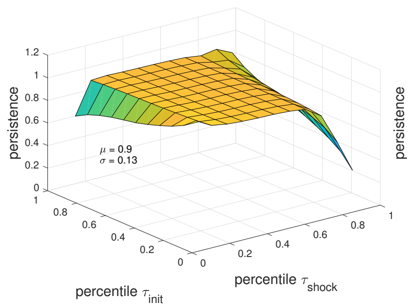
Figure G5: Joint density of firm heterogeneity and shocks, Sector 10



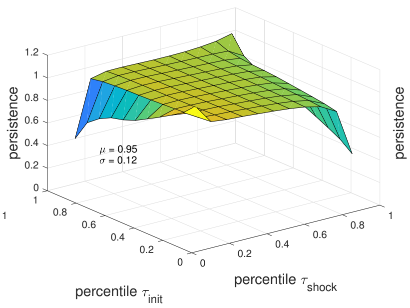
Notes: Sector 25, metal products. Panels (a)–(c) are contour plots of the estimated joint distributions of productivity and demand advantages, productivity and labor market advantages, and demand and labor market advantages, respectively. Panels (d)–(f) are contours of the estimated copula densities of the shocks to productivity (v^{ω}_{jt}), demand (v^{δ}_{jt}), and labor market advantages (v^{S}_{jt}). As a graphical convention, I rescale the marginals of the shocks so they are standard normal. Correlations also reported.

Figure G6: Dynamics of productivity, demand, and labor market advantages, Sector 10

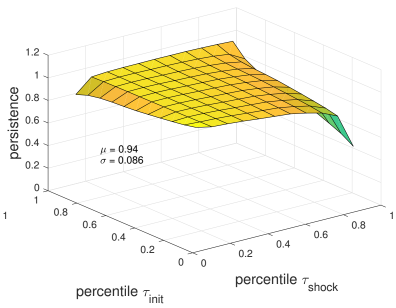
(a) Conditional persistence in productivity



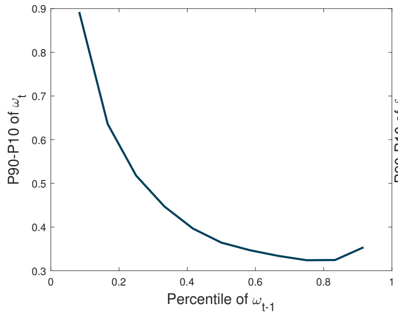
(b) Conditional persistence in demand advantages



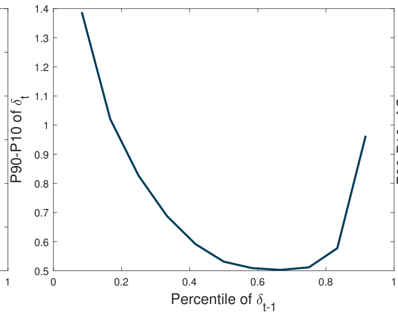
(c) Conditional persistence in labor market advantages



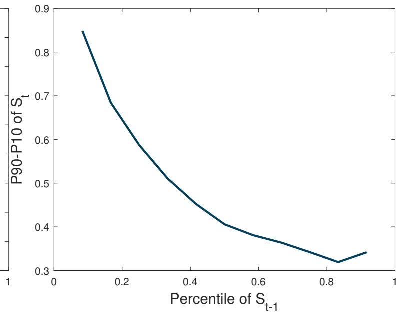
(d) Conditional dispersion of productivity



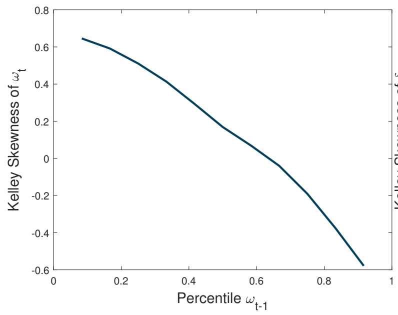
(e) Conditional dispersion of demand advantages



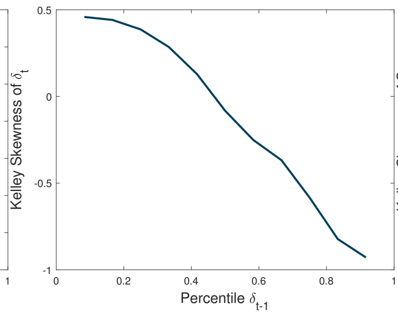
(f) Conditional dispersion of labor market advantages



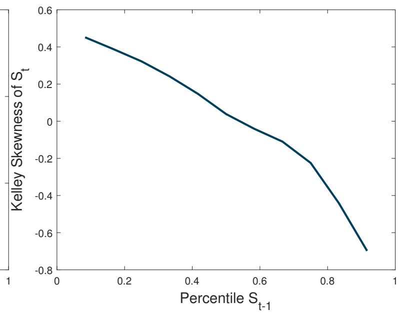
(g) Conditional skeweness of productivity



(h) Conditional skewness of demand advantages



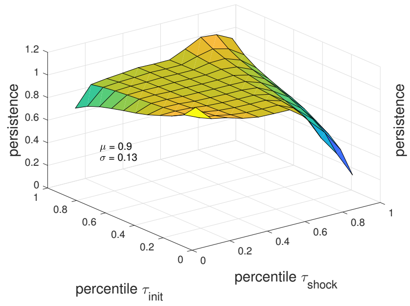
(i) Conditional skewness of labor market advantages



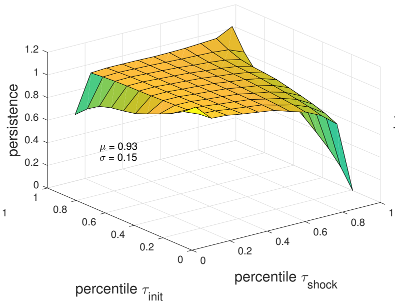
Notes: Sector 10, food products. Panels (a)–(c) are estimates of the persistence of productivity, demand, and labor market advantages, respectively, conditional on the percentile of the past state (τ_{init}) and percentile of the shock (τ_{shock}). They are obtained as estimates of the average derivative of the conditional quantile function of the state z_{jt} given the previous state z_{jt-1} with respect to z_{jt-1}. Panels (d)–(f) present estimates of the conditional dispersion of the states given past states measured as the $P90 - P10$ of the predictive distribution. Panel (g)–(j) present estimates of the conditional skewness of the states given past states measured as $\frac{(P90-P50)-(P50-P10)}{P90-p10}$ of the predictive distribution.

Figure G7: Dynamics of productivity, demand, and labor market advantages, Sector 14

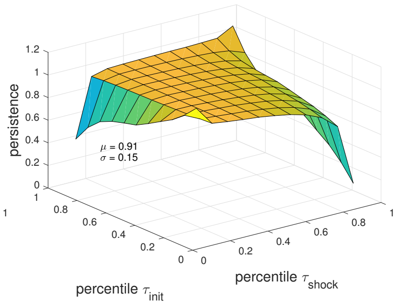
(a) Conditional persistence in productivity



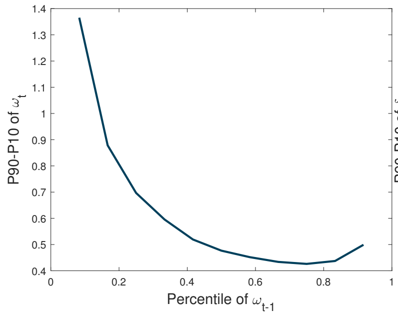
(b) Conditional persistence in demand advantages



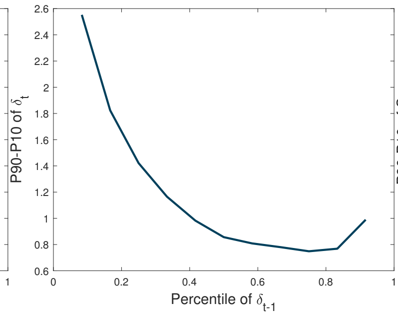
(c) Conditional persistence in labor market advantages



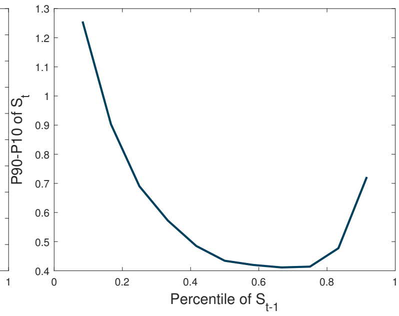
(d) Conditional dispersion of productivity



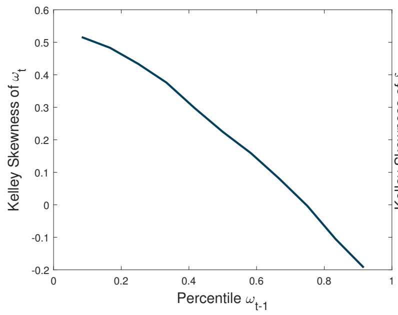
(e) Conditional dispersion of demand advantages



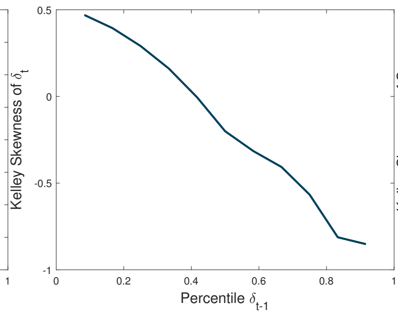
(f) Conditional dispersion of labor market advantages



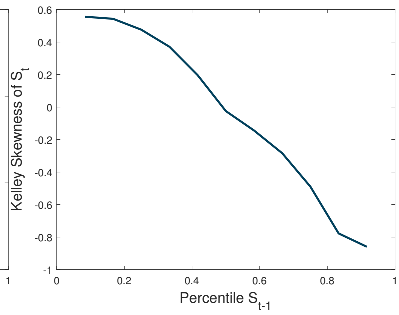
(g) Conditional skeweness of productivity



(h) Conditional skewness of demand advantages



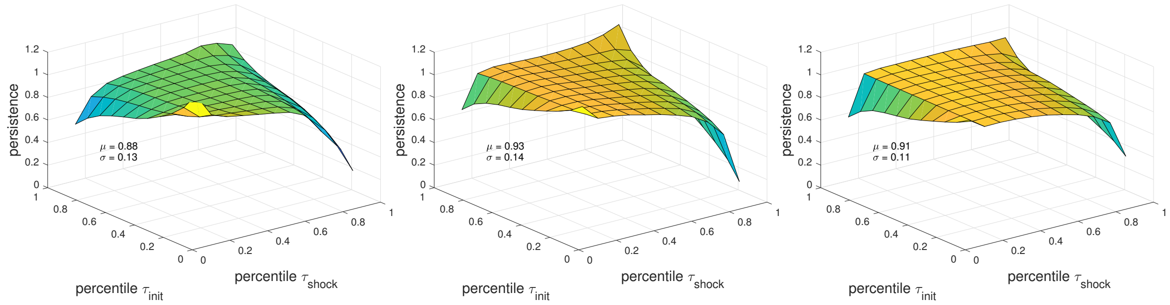
(i) Conditional skewness of labor market advantages



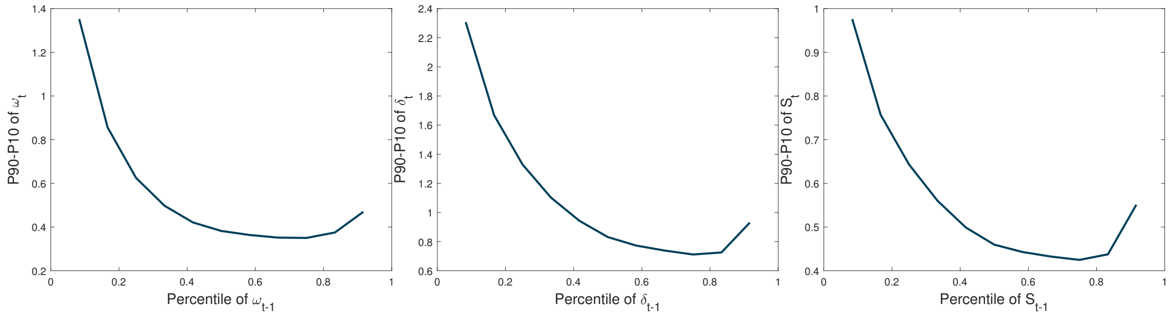
Notes: Sector 14, clothing. Panels (a)–(c) are estimates of the persistence of productivity, demand, and labor market advantages, respectively, conditional on the percentile of the past state (τ_{init}) and percentile of the shock (τ_{shock}). They are obtained as estimates of the average derivative of the conditional quantile function of the state z_{jt} given the previous state z_{jt-1} with respect to z_{jt-1}. Panels (d)–(f) present estimates of the conditional dispersion of the states given past states measured as the $P90 - P10$ of the predictive distribution. Panel (g)–(j) present estimates of the conditional skewness of the states given past states measured as $\frac{(P90-P50)-(P50-P10)}{P90-p10}$ of the predictive distribution.

Figure G8: Dynamics of productivity, demand, and labor market advantages, Sector 25

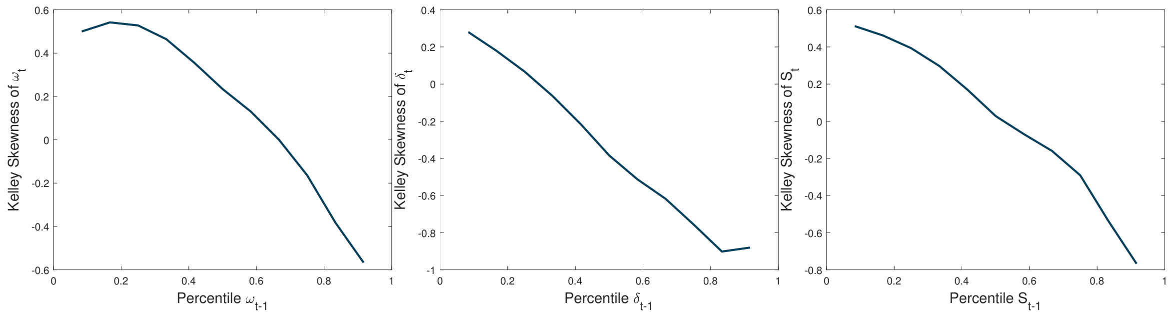
(a) Conditional persistence in productivity (b) Conditional persistence in demand advantages (c) Conditional persistence in labor market advantages



(d) Conditional dispersion of productivity (e) Conditional dispersion of demand advantages (f) Conditional dispersion of labor market advantages

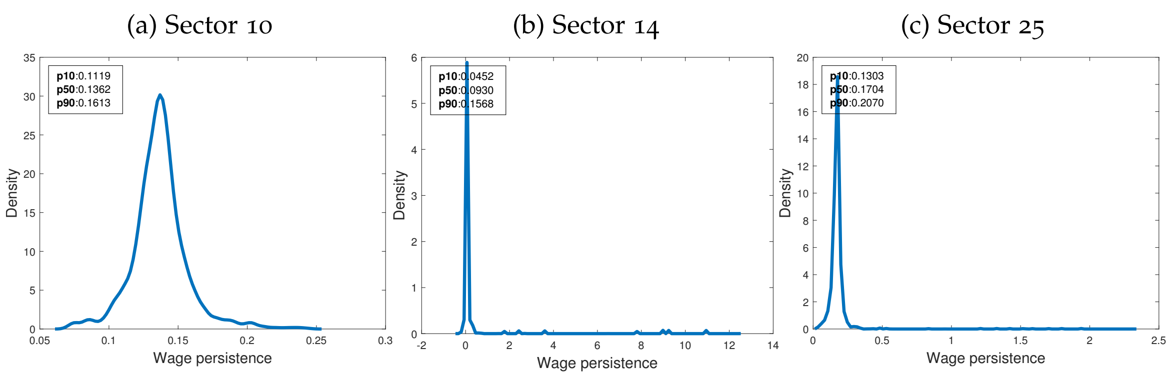


(g) Conditional skeweness of productivity (h) Conditional skewness of demand advantages (i) Conditional skewness of labor market advantages



Notes: Sector 25, metal products. Panels (a)–(c) are estimates of the persistence of productivity, demand, and labor market advantages, respectively, conditional on the percentile of the past state (τ_{init}) and percentile of the shock (τ_{shock}). They are obtained as estimates of the average derivative of the conditional quantile function of the state z_{jt} given the previous state z_{jt-1} with respect to z_{jt-1}. Panels (d)–(f) present estimates of the conditional dispersion of the states given past states measured as the $P90 - P10$ of the predictive distribution. Panel (g)–(j) present estimates of the conditional skewness of the states given past states measured as $\frac{(P90-P50)-(P50-P10)}{P90-p10}$ of the predictive distribution.

Figure G9: Autocorrelation in wages, by sector



Notes: Marginal distribution of the derivative of the wage-setting equation given past log-wages $\ln w_{jt-1}$ *and other state variables (capital* k_{jt}*, productivity* ω_{jt}*, demand* δ_{jt}*, and labor market advantages* S_{jt}*) with respect to* $\ln w_{jt-1}$*. Distributions of individual sectors weighted by sector sales. Kernel densities estimated on data with top and bottom 2% trimmed. Percentiles computed with full data. Sector 10 is food products, Sector 14 is clothing, and Sector 25 is metal products.*