Models of Wage Determination with Two-Sided Heterogeneity Using Matched Employer-Employee Data

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Motivation: Sources of wage inequality

How much of wage inequality can we attribute to...

- **Worker heterogeneity**: human capital, discrimination
- **Firm heterogeneity**: search and matching frictions + labor market power
- **Sorting**: production complementarities

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Workhorse model: AKM two-way fixed effects (Abowd et al., 1999)

$$\ln w_{it} = x_{it}'\beta + \alpha_i + \psi_{j(i,t)} + \varepsilon_{it}$$

with particular quantities of interest:

- Variance of firm effects $\rightarrow \text{Var}(\psi_{j(i,t)})$
- Sorting $\rightarrow \text{Cov}(\alpha_i, \psi_{j(i,t)})$

What have we learned?

Lessons from previous work using the two-way FE model:

- Sizable role of firm FE (typically explaining 20% of wage variation)
- Correlation of firm and worker FE are small indicating little to no sorting
- Sorting has been increasing over time
- Between-firm wage inequality has been increasing

What have we learned?

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How reliable are these conclusions?

- FE estimators suffer from an incidental parameter bias ("limited mobility bias")
- Variance of firm FE upward biased
- Covariance of worker and firm FE downward biased

Outline

AKM two-way FE model and the limited mobility bias

Random effects approach

Discretized heterogeneity + hybrid FE-RE approach

- Digression: Grouped fixed effects (GFE)
- Bonhomme, Lamadon, and Manresa (2019)
- Related work

Outline

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AKM model

$$\ln w_{it} = x_{it}'\beta + \alpha_i + \psi_{j(i,t)} + \varepsilon_{it}$$

- The model is static in two senses:
  - Firm and worker effects are time-invariant
  - Wages do not depend on the past $\rightarrow$ inconsistent with models of on-the-job search or wage adjustment costs
- Worker and firm effects may not map to structural objects (Eeckhout and Kircher, 2011) $\rightarrow$ thought it remains a useful reduced-form tool
- To make progress, we typically assume exogenous mobility:

$$\mathbb{E}(\varepsilon_{it} \mid X, D; \alpha, \psi) = 0$$

  which precludes mobility that depends on match effects, for instance.

Estimation and identification

AKM least-squares estimator

Under exogenous mobility, the least squares estimator provides us

- Consistent estimator for β
- $(\widehat{\alpha}, \widehat{\psi})$ are unbiased but not fixed-T consistent
- Empirical counterparts for variance decomposition: $\mathrm{Var}(\widehat{\psi}_j), \mathrm{Cov}(\widehat{\alpha}_i, \widehat{\psi}_{j(i,t)})$

Computational implementation not as straightforward (Abowd et al., 1999, 2002; Guimarães and Portugal, 2010)

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Identification of firm effects by movers

The differences in firm effects are identified by movers:

$$\mathbb{E}(Y_{it+1} - Y_{it} \mid j(i,t) = j, j(i,t+1) = j') = \psi_{j'} - \psi_j$$

Need a connected set and one normalization

Limited mobility bias: An illustration

Case I: 1 mover

$$Y_{i1} = \alpha_i + \qquad + \varepsilon_{i1}$$
$$Y_{i2} = \alpha_i + \psi_j + \varepsilon_{i2}$$

- Estimator for ψ_j:

$$\widehat{\psi}_j = Y_{i2} - Y_{i1} = \psi_j + (\varepsilon_{i2} - \varepsilon_{i1})$$
$$\Rightarrow \text{Var}(\widehat{\psi}_j) = \text{Var}(\psi_j) + 2\text{Var}(\varepsilon)$$

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$$\widehat{\alpha}_i = Y_{i2} - \widehat{\psi}_j = \underbrace{(Y_{i2} - \psi_j)}_{=\alpha_i + \varepsilon_{i2}} + \varepsilon_{i1} - \varepsilon_{i2} = \alpha_i + \varepsilon_{i1}$$

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- Estimator for covariance:

$$\text{Cov}(\widehat{\alpha}_i, \widehat{\psi}_j) = \text{Cov}(\alpha_i + \varepsilon_{i1}, \psi_j + (\varepsilon_{i2} - \varepsilon_{i1})) = \text{Cov}(\alpha_i, \psi_j) - \text{Var}(\varepsilon)$$

Limited mobility bias: An illustration

Case II: 2 movers

$$
\begin{aligned}
Y_{i1} &= \alpha_i + + \varepsilon_{i1} & Y_{i'1} &= \alpha_{i'} + + \varepsilon_{i'1} \\
Y_{i2} &= \alpha_i + \psi_j + \varepsilon_{i2} & Y_{i'2} &= \alpha_{i'} + \psi_j + \varepsilon_{i'2}
\end{aligned}
$$

- Estimator for ψ_j:

$$
\widehat{\psi}_j = \frac{(Y_{i2} - Y_{i1}) + (Y_{i'2} - Y_{i'1})}{2} = \psi_j + \frac{(\varepsilon_{i2} - \varepsilon_{i1}) + (\varepsilon_{i'2} - \varepsilon_{i'1})}{2}
$$

$$
\Rightarrow \text{Var}(\widehat{\psi}_j) = \text{Var}(\psi_j) + \text{Var}(\varepsilon)
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Limited mobility bias: An illustration

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- Estimator for α_i:

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$$

Limited mobility bias: An illustration

Case II: 2 movers

$$\begin{aligned} Y_{i1} &= \alpha_i + \quad\;\; + \varepsilon_{i1} & Y_{i'1} &= \alpha_{i'} + \quad\;\; + \varepsilon_{i'1} \\ Y_{i2} &= \alpha_i + \psi_j + \varepsilon_{i2} & Y_{i'2} &= \alpha_{i'} + \psi_j + \varepsilon_{i'2} \end{aligned}$$

- Estimator for ψ_j:

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$$\Rightarrow \text{Var}(\widehat{\psi}_j) = \text{Var}(\psi_j) + \text{Var}(\varepsilon)$$

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$$\widehat{\alpha}_i = Y_{i2} - \widehat{\psi}_j = \underbrace{(Y_{i2} - \psi_j)}_{=\alpha_i + \varepsilon_{i2}} - \frac{(\varepsilon_{i2} - \varepsilon_{i1}) + (\varepsilon_{i'2} - \varepsilon_{i'1})}{2} = \alpha_i + \frac{(\varepsilon_{i1} + \varepsilon_{i2}) + (\varepsilon_{i'1} - \varepsilon_{i'2})}{2}$$

- Estimator for covariance:

$$\text{Cov}(\widehat{\alpha}_i, \widehat{\psi}_j) = \text{Cov}(\alpha_i, \psi_j) - \frac{1}{2}\text{Var}(\varepsilon)$$

Limited mobility bias: An illustration

Comparison

Variance component	1 mover	2 movers
$\text{Var}(\widehat{\psi}_j)$	$\text{Var}(\psi_j) + 2\text{Var}(\varepsilon)$	$\text{Var}(\psi_j) + \text{Var}(\varepsilon)$
$\text{Cov}(\widehat{\alpha}_i, \widehat{\psi}_j)$	$\text{Cov}(\alpha_i, \psi_j) - \text{Var}(\varepsilon)$	$\text{Cov}(\alpha_i, \psi_j) - \frac{1}{2}\text{Var}(\varepsilon)$

- The contribution of firm effect on the variance of wages biased upward
- Sorting (covariance) of worker and firm effects biased downward (may even reverse sign)
- Size of bias depends on number of movers

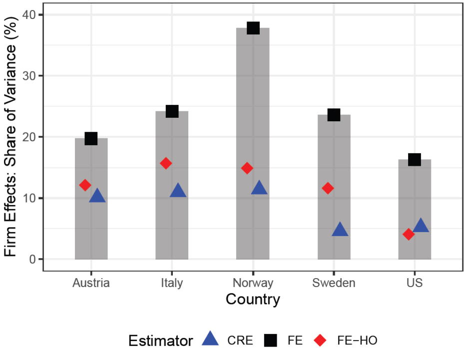
Bias-correction of variance components

- Andrews et al. (2008) provides a characterization of the bias in the variance components based on the AKM estimators and an bias-correction **under homoskedasticity**
- Kline et al. (2020) propose a jack-knife based bias-correction **under heteroskedasticity**
- For large networks, the exact bias-correction estimators are computationally infeasible as they involve inverting large matrices $\rightarrow$ computationally-feasible approximations proposed

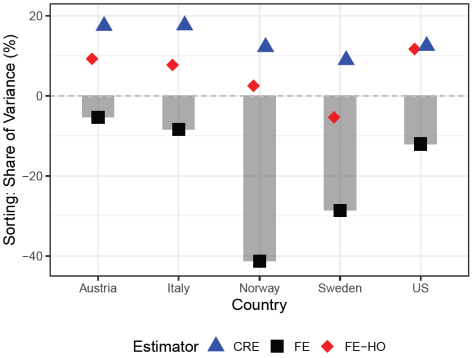
Bias-corrected estimates across countries

Bonhomme et al. (2023)

(b) Firm Effects (3-year panel)



(d) Sorting (3-year panel)



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Random effect approaches

- Fixed effects approaches are attractive because we can be agnostic about the unobserved heterogeneity $\rightarrow$ α's and ψ's left unrestricted
- Random effects approaches require us to model the unobserved heterogeneity
  - Augment with a model of the joint distribution of $(\alpha, \psi) \mid D$
  - Made to depend on a smaller number of parameters $\rightarrow$ computational tractability and more precise estimates

RE specification by Woodcock (2008)

$$\begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_N \\ \psi_1 \\ \vdots \\ \psi_J \end{pmatrix} \mid D \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\alpha^2 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_\alpha^2 & 0 & \dots & 0 \\ 0 & \dots & 0 & \sigma_\psi^2 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & \dots & \sigma_\psi^2 \end{pmatrix} \right)$$

Assumptions quite strong:

- No sorting
- Cannot capture "workers working in same firm are similar" or "firms that employ the same workers are similar"
- Woodcock (2008) provide a specification with match effects that may relax some of the economic assumptions

“Restricted ML” estimator by Woodcock (2008)

Take MLE $(\tilde{\alpha}, \tilde{\psi}, \tilde{\sigma}^2_\alpha, \tilde{\sigma}^2_\psi, \tilde{\sigma}^2_\varepsilon)$ corresponding to the likelihood

$$\begin{aligned}\log f(Y, \alpha, \psi \mid D) &= \log f(Y \mid D, \alpha, \psi) + \log f(\alpha, \psi \mid D) \\ &= -\frac{1}{2}\log \sigma^2_\varepsilon - \frac{1}{2\sigma^2_\varepsilon}(Y - D\gamma)'(Y - D\gamma) \\ &\quad - \frac{1}{2}\log \mid \Sigma_{(\sigma^2_\alpha, \sigma^2_\psi)} \mid - \frac{1}{2}(\alpha', \psi')\Sigma^{-1}_{(\sigma^2_\alpha, \sigma^2_\psi)}(\alpha', \psi')'\end{aligned}$$

- $(\tilde{\alpha}, \tilde{\psi})$ can be shown to be posterior mean estimates of (α, ψ) viewing our specified model of $(\alpha, \psi) \mid D$ as a prior (Abowd et al., 2008)
- $(\tilde{\alpha}, \tilde{\psi})$ can be seen as “shrinkage” estimates (shrinking towards a model with fully-random matching)
- Consistency still relies on correct specification

Related work

- Random effects approach may open some possibilities:
  - Dynamics in firm and worker effects
  - Complex Markovian structures: persistent-transitory components
- Related papers: Friedrich et al. (2021); Bingley and Cappellari (2022)

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Grouped fixed effects

- In practice, we face a trade-off between
  - Modeling unobserved heterogeneity flexibly
  - Keeping parsimonious specifications for the observed data
- Models with discretized heterogeneity is an attempt to resolve the tradeoff
- Especially in nonlinear models, GFE may alleviate incidental parameter biases
- Example: time-varying group effects + group-specific coefficients

$$y_{it} = x_{it}'\theta_{g_i} + \alpha_{g_i,t} + \varepsilon_{it}$$

  with group membership variables $g_i \in \{1, ..., G\}$
- In environments where unobserved heterogeneity may be continuous, GFE can be thought of as regularization or dimension reduction (Bonhomme et al., 2021)

GFE estimation

Iterative as in Bonhomme and Manresa (2015)

$$(\widehat{\theta}, \widehat{\alpha}, \widehat{\gamma}) = \underset{\theta,\alpha,\gamma}{\operatorname{argmin}} \sum_{i=1}^{N} \sum_{t=1}^{T} (y_{it} - x_{it}'\theta_{g_i} - \alpha_{g_i,t})^2$$

We can solve this using an iterative algorithm. Start with initial guess $(\theta^{(0)}, \alpha^{(0)})$

1. **Assignment.** For $i = 1, ..., N$

$$g_i^{(s+1)} = \underset{g \in \{1,...,G\}}{\operatorname{argmin}} \sum_{t=1}^{T} (y_{it} - x_{it}'\theta_g^{(s)} - \alpha_{g,t}^{(s)})^2$$

2. **Update.**

$$(\theta^{(s+1)}, \alpha^{(s+1)}) = \underset{\theta,\alpha}{\operatorname{argmin}} \sum_{i=1}^{N} \sum_{t=1}^{T} \left(y_{it} - x_{it}'\theta_{g_i^{(s+1)}} - \alpha_{g_i^{(s+1)},t} \right)^2$$

3. Iterate between Steps 1 and 2 until numerical convergence

Other algorithms can be considered to improve speed and accuracy

GFE estimation

Two-step as in Bonhomme et al. (2021)

- In some models, the updating step is computationally costly
- We might have access to moments of the data, h_i, that are informative of the underlying unobserved heterogeneity $\rightarrow$ two-step estimation
- Example: $y_{it} = \alpha_{g_i} + \varepsilon_{it}$ then $\overline{y}_i$ informative of α_{g_i}
- **Step 1 (Classification).** Fix G. Start with initial guess of means $\widetilde{h}_1^{(0)}, ..., \widetilde{h}_G^{(0)}$
  1. **Assignment.** For $i = 1, ..., N$

$$g_i^{(s+1)} = \underset{g=1,\dots,G}{\operatorname{argmin}} \|h_i - \widetilde{h}_g^{(s)}\|^2$$

  2. **Update.**

$$\widetilde{h}_g^{(s+1)} = \frac{1}{\#\{i : g_i^{(s+1)} = g\}} \sum_{\{i:g_i^{(s+1)}=g\}} h_i$$

- **Step 2.** Estimate parameters of model conditional on groups from Step 1

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Model set-up

Static model of Bonhomme et al. (2019)

- Worker types: $\ell(i) \in \{1, ..., L\}$
- Firm classes: $k(j) \in \{1, ..., K\}$
- Treat firm classes as FE and worker types as RE
- Restrictions:

$$
\begin{aligned}
f(Y_1, Y_2, k' \mid k) &= \sum_{\ell=1}^{L} f(Y_2 \mid Y_1, k, k', \ell) \times f(k' \mid Y_1, k, \ell) \times f(Y_1 \mid k, \ell) \times f(\ell \mid k) \\
&= \sum_{\ell=1}^{L} f(Y_2 \mid \qquad k', \ell) \times f(k' \mid \qquad k, \ell) \times f(Y_1 \mid k, \ell) \times f(\ell \mid k)
\end{aligned}
$$

- Two-stage estimation:
  1. Recover firm classes
  2. Estimate parameters conditional on estimated firm classes by ML

Recovering firm classes

- Note that the wage distribution of a firm in period 1 is

$$\Pr(Y_1 \leq y \mid j) = \sum_{\ell=1}^{L} \Pr(Y_1 \leq y \mid k(j), \ell) \times \Pr(\ell \mid k(j))$$

which only depends on the firm's class $k(j)$

- This motivates that we may recover firm classes by clustering based on the firm's wage distribution

$$\min_{k(1),\ldots,k(J),H_1,\ldots,H_K} \sum_{j=1}^{J} n_j \sum_{d=1}^{D} \left(\widehat{F}_j(y_d) - H_{k(j)}(y_d) \right)^2$$

i.e., k-means clustering on D CDF points

Identifying complementarities in wages

- To gain some intuition, consider the following model of wages

$$Y_{it}(k_{it}) = a(k_{it}) + b(k_{it}) \times \alpha(\ell_i) + \varepsilon_{it}$$

- Consider movers from two firm classes:

$$\overline{y}_{2\to 1}(2) - \overline{y}_{1\to 2}(2) = b(2)(\overline{\alpha}_{2\to 1} - \overline{\alpha}_{1\to 2})$$

$$\overline{y}_{2\to 1}(1) - \overline{y}_{1\to 2}(1) = b(1)(\overline{\alpha}_{2\to 1} - \overline{\alpha}_{1\to 2})$$

  then the ratio identifies $\frac{b(2)}{b(1)}$

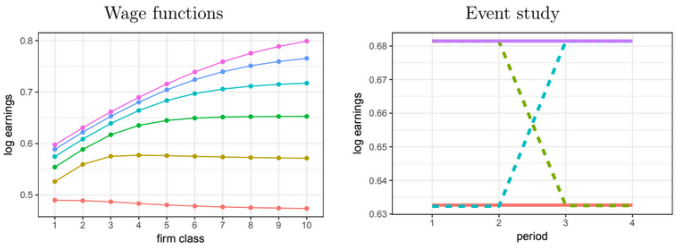
- Identification is based on movers between *firm classes*
  - We need movers in both directions (cycles) $\to$ by every worker type
  - We need differential mobility ($\overline{\alpha}_{2\to 1} \neq \overline{\alpha}_{1\to 2}$)

What can we learn from mean restrictions?

- Note that

$$(\overline{y}_{2\to 1}(2) - \overline{y}_{2\to 1}(1)) + (\overline{y}_{1\to 2}(1) - \overline{y}_{1\to 2}(2)) = (b(2) - b(1))(\overline{\alpha}_{2\to 1} - \overline{\alpha}_{1\to 2})$$

  which is zero if $b(2) - b(1) = 0$ or $\overline{\alpha}_{2\to 1} = \overline{\alpha}_{1\to 2}$
- Looking at wage changes between $1 \to 2$ and $2 \to 1$ is not necessarily a test of linearity



Notes: The sample is generated according to the model of Shimer and Smith (2000), without on the job search. The parameter values imply positive assortative matching. In the left graph we show log-wage functions for each worker type (y-axis), by firm class (x-axis). In the right graph we show mean log-wages of workers moving between firms within classes 4 and 10 (solid), and moving between firms between classes 4 and 10 (dashed), between periods 2 and 3.

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Related work

- Abowd et al. (2019) models match effects and mobility to quantify biases induced by endogenous mobility
- Lentz et al. (forthcoming)
  - Augments the model with a structural matching model (with unemployment) to study how job preferences, layoffs, market segmentation, and reemployment affect mobility and sorting
  - Nest the EM with a classification step (iterative algorithm classification + parameter updating)

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